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# What exactly are the double PDFs?

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# Outline

Ever since the first papers discussing double parton scattering (DPS) were published, popular opinion has been that pp DPS cross sections may be reasonably described by the following formula:

'Double PDFs'

$$\sigma_D^{(A,B)} = \frac{1}{\sigma_{eff}} \sum_{p_1 p_2 p_3 p_4} \int \mathrm{d}x_1 \, \mathrm{d}x_1' \mathrm{d}x_2 \, \mathrm{d}x_2' D_h^{p_1 p_2}(x_1, x_2; Q_A, Q_B) D_h^{p_3 p_4}(x_1', x_2'; Q_A, Q_B) \times \text{hard part} \quad (1)$$

Double DGLAP equation exists which supposedly dictates the evolution of the equal scale dPDFs → we used this to produce GS09 dPDFs.

However, some recent studies have suggested that actually one should use 2pGPDs with longitudinal momentum + transverse separation arguments to describe pp DPS.

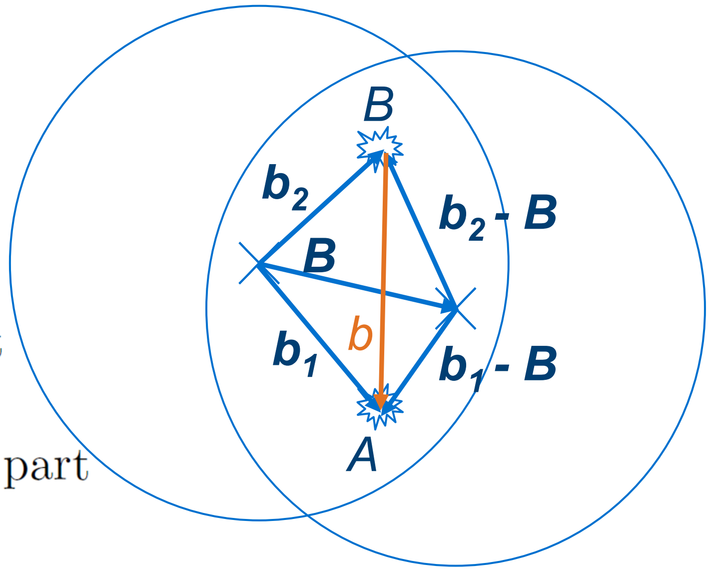
Does the concept of a double PDF have any meaning? If so, can we probe double PDFs experimentally? How wrong is it to use (1) & GS09 dPDFs to calculate DPS cross sections? I will attempt to answer these questions in this talk.

# DPS cross section

$$\sigma^D_{A,B} = \int d^2\boldsymbol{b_1} d^2\boldsymbol{b_2} d^2\boldsymbol{B}\, D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_2) D_h^{p_3p_4}(x'_1,x'_2;\boldsymbol{b}_1-\boldsymbol{B},\boldsymbol{b}_2-\boldsymbol{B}) \times \text{hard part}$$

Geometrical picture:

Changing variables:




$$\sigma^D_{A,B} = \int d^2\boldsymbol{b} \int d^2\boldsymbol{b_1} D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_1-\boldsymbol{b})$$
$$\times \int d^2\boldsymbol{B'} D_h^{p_3p_4}(x'_1,x'_2;\boldsymbol{B'},\boldsymbol{B'}-\boldsymbol{b}) \times \text{hard part}$$
$$= \int d^2\boldsymbol{b} D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}) D_h^{p_3p_4}(x'_1,x'_2;\boldsymbol{b}) \times \text{hard part}$$

where: $D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}) = \int d^2\boldsymbol{b}_1 d^2\boldsymbol{b}_2 D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_2)\delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_1+\boldsymbol{b}_2)$ ‘2pGPD’

Not enough transverse momentum integrations compared to constraints to write the DPS cross section in terms of fully integrated PDFs. Related to the fact that parton pairs from both protons must be separated by the same amount for double interaction.

# The factorisation assumption

In almost all studies of DPS: Assume that 2pGPD may be taken as a product of a longitudinal and a (typically flavour independent) transverse piece.

$$D_h^{p1p2}(x_1, x_2; \boldsymbol{b}) = D_h^{p1p2}(x_1, x_2) F(\boldsymbol{b})$$

Then the DPS cross section reduces to the familiar form:

$$\sigma_D^{(A,B)} = \frac{1}{\sigma_{eff}} \sum_{p_1 p_2 p_3 p_4} \int \mathrm{d}x_1 \, \mathrm{d}x_1' \mathrm{d}x_2 \, \mathrm{d}x_2' D_h^{p_1 p_2}\left(x_1, x_2; Q_A, Q_B\right) D_h^{p_3 p_4}\left(x_1', x_2'; Q_A, Q_B\right)$$

$$\times \,\text{hard part}$$

where: $\sigma_{eff} = \dfrac{1}{\int \left[F(\mathbf{b})\right]^2 d^2\mathbf{b}}$ 'Effective area'

# Double DGLAP equation

An equation dictating the scaling violations of a quantity denoted as $D_h^{j1j2}(x_1,x_2,Q^2)$ was derived in 1982 by Shelest, Snigirev and Zinovjev [Phys. Lett. B 113:325]. Several papers since [e.g. Snigirev, hep-ph/0304172] have suggested that this quantity is equal to the factorised longitudinal piece of the 2pGPD for the case where $Q_A = Q_B \equiv Q$.

$$\begin{aligned}
\frac{dD_h^{j_1 j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\Bigg[&\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1' j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right) \\
&+\sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1 j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right) \\
&+\sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1 j_2}\left(\frac{x_1}{x_1+x_2}\right)\Bigg]
\end{aligned}$$

Usual 1→1 splitting functions

$$\boxed{t = \ln\left(Q^2\right)}$$

'1→2' splitting function

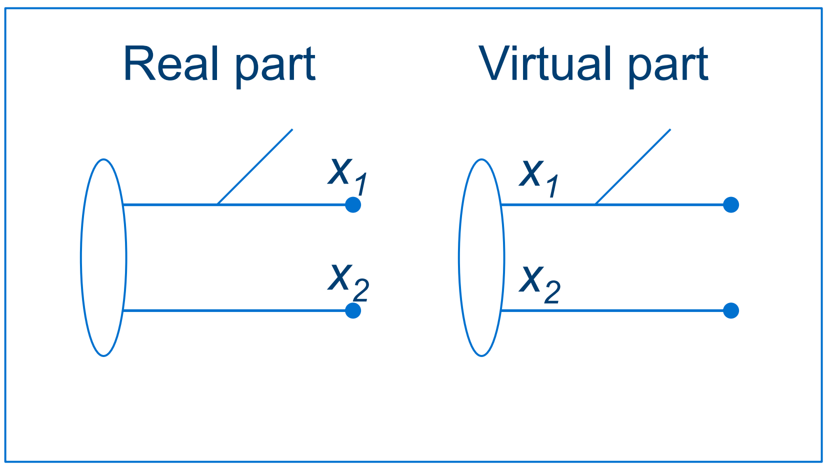
Single PDF

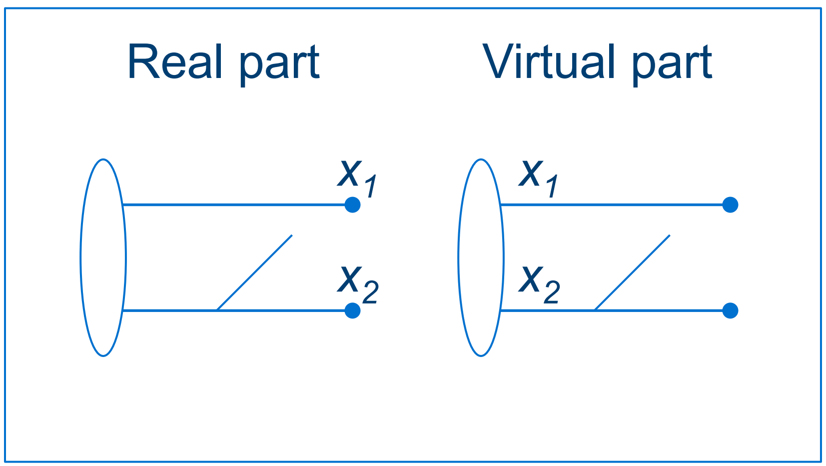
[Structure of last term must be altered at NLO and above]

# Pictorial Representation of the dDGLAP equation

$$\frac{dD_h^{j_1 j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\left[\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1' j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right) + \sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1 j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right) + \sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1 j_2}\left(\frac{x_1}{x_1+x_2}\right)\right]$$

'Independent branching' terms



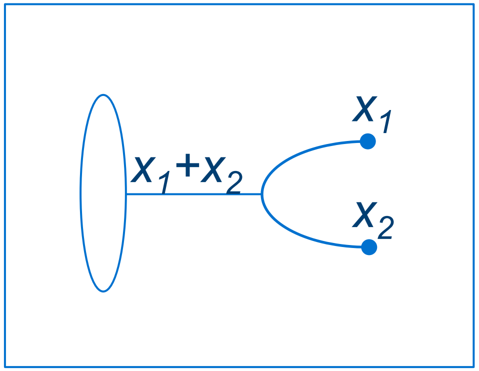
# Pictorial Representation of the dDGLAP equation

$$\frac{dD_h^{j_1j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\left[\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1'j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right)\right.$$

$$+\sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right)$$

$$\left.\boxed{+\sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1j_2}\left(\frac{x_1}{x_1+x_2}\right)}\right]$$

'sPDF feed' term



# The GS09 dPDFs

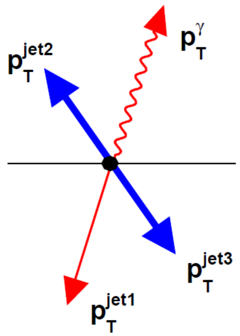
JG and Stirling, 0910.4347, 2009

A set of LO equal-scale double PDFs (dPDFs) – by equal-scale dPDF we mean the quantity that appears in the dDGLAP equation (whatever that is).

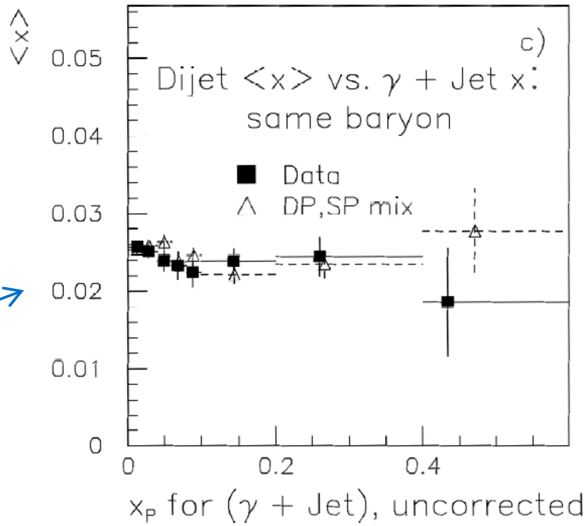
dDGLAP equation only tells us how dPDFs change with scale, not what they are at any particular scale → need to construct some sensible input functions

$$D_h^{p_1 p_2}\left(x_1, x_2; Q_0^2\right)$$

We used two pieces of information to guide construction of inputs:




1) CDF analysis of γ+3jet DPS events – *x* of parton taking part in dijet production is not strongly correlated with that taking part in γ+jet production.




Phys. Rev. D56 3811–3832.

# The GS09 dPDFs

JG and Stirling, 0910.4347, 2009

2) The dDGLAP equation preserves the following "sum rule" equalities, provided that they hold at the initial scale:

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 D_h^{j_1 j_2}(x_1, x_2; t) = (1 - x_2) D_h^{j_2}(x_2; t)$$

$$\int_0^{1-x_2} dx_1 D_h^{j_{1v} j_2}(x_1, x_2; t) = \begin{cases} N_{j_{1v}} D_h^{j_2}(x_2; t) & \text{when } j_2 \neq j_1 \text{ or } \bar{j}_1 \\ (N_{j_{1v}} - 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = j_1 \\ (N_{j_{1v}} + 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = \bar{j}_1 \end{cases}$$

We interpreted these relations as the momentum and number sum rules for the dPDFs using simple conditional probability arguments, and suggested therefore that they should hold at the initial scale.

# The GS09 dPDFs

JG and Stirling, 0910.4347, 2009

GS09 initial scale dPDFs at $Q_0$ = 1 GeV are simple products of MSTW2008LO single PDFs, appropriately modified (using additional factors/extra terms) such that the sum rules are approximately satisfied.

dPDFs at other scales are then obtained by evolving these inputs using the LO dDGLAP equation.

The publicly available grid files cover the range $10^{-6} < x_1 < 1$, $10^{-6} < x_2 < 1$, and $1\ \mathrm{GeV}^2 < Q^2 < 10^9\ \mathrm{GeV}^2$.

# Breakdown of the factorisation assumption

Several groups working on DPS (Diehl et al., Blok et al.) are now of the opinion that the 2pGPD may **not** be factorised into longitudinal and transverse pieces – we agree with this.

At the heart of this issue is the 'sPDF feed' term of the dDGLAP equation just mentioned. If the 2pGPD can be factorised into longitudinal + transverse pieces, we expect 2pGPD to obey double DGLAP equation for any ***b***.

However, for nonzero ***b***, there is no way that the evolution equation for the 2pGPD can contain the sPDF term [Diehl, talk at MPI 2010 at DESY].

To illustrate this let us consider the leading contribution to the distribution of quark-antiquark pairs inside a gluon. The ***b*** space quark-antiquark wavefunction of the dressed gluon state goes like $1/b$. Thus:

$$D_g^{q\bar{q}}(b) \propto \left|\Psi(b)\right|^2 \propto \frac{1}{b^2}$$

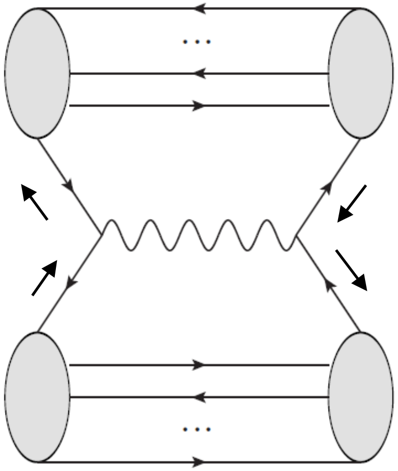
# Breakdown of the factorisation assumption

$$D_g^{q\bar{q}}(b) \propto \left|\Psi(b)\right|^2 \propto \frac{1}{b^2}$$

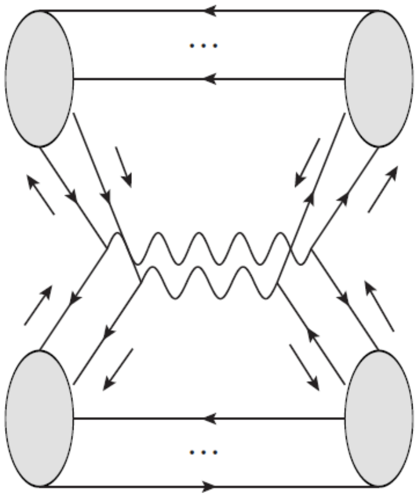
In order to generate a logarithmic singularity in $D_g^{q\bar{q}}$ , the coefficient of which we could identify with a $g \to q\bar{q}$ sPDF feed term, we must integrate over ***b***.

If we fix ***b*** between the active partons at some finite value then there are no leading log singularities associated with a parton splitting into the two active partons → no sPDF feed term in the evolution equation.

# Interference contributions to proton-proton DPS



In proton-proton SPS, only one parton leaves each proton, interacts, and then returns
→ interacting parton must return with the same quantum numbers as it left with such that it can recombine with spectators to form original proton
→ No interference contribution to proton-proton SPS



In proton-proton DPS, fact that interacting partons must recombine with spectators to form original proton only imposes conditions on ‘sum’ of quantum numbers of active partons
→ Possibility of interference diagrams in which flavour, spin or colour are swapped between active partons, provided that a swap in the opposite direction occurs for the other proton.

# Polarised PDF contributions to proton-proton DPS

In proton-proton DPS, there exists the possibility of having contributions to the cross section associated with polarized 2pGPDs, *even when the colliding protons are unpolarized*!

Reason for this: there may be correlations in helicity *between* the two active partons!

e.g. $$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

If probability to find two quarks with same spin differs from probability to find two quarks with opposing spins, $\Delta q_1 \Delta q_2 \neq 0$.

Similarly – contributions associated with colour correlations between partons.

Issues of interference & spin/colour correlations previously raised by Diehl (see e.g. his talk at DESY MPI workshop this year).

# What is a dPDF?

Does the concept of a ‘double PDF’ with only *x* arguments have any meaning?

Yes – we can just define the dPDF as the integral of the 2pGPD over ***b*** (restrict ourselves to the case $Q_A = Q_B = Q$ in this talk):

$$D_h^{p_1 p_2}(x_1, x_2; Q) \equiv \int d^2\mathbf{b} D_h^{p_1 p_2}(x_1, x_2, \mathbf{b}; Q, Q)$$

Light-cone expansion of (unpolarized) dPDF:

‘Boost invariant’
LC wavefunction

$$D_h^{p_1 p_2}(x, y; Q^2) = \sum_{N,\beta} \int [dx]_N \int^Q [d^2\boldsymbol{k}]_N |\Phi_N(\beta, \{x_i, \boldsymbol{k}_i\})|^2 \sum_i^N \sum_{j\neq i}^N \delta(x - x_i)\delta(y - y_j)\delta_{p_i p_1}\delta_{p_j p_2}$$

$$[dx]_N \equiv \prod_{i=1}^N dx_i \delta\left(1 - \sum_i^N x_i\right) \qquad [d^2\boldsymbol{k}]_N \equiv \prod_{i=1}^N d^2\boldsymbol{k}_i \delta^{(2)}(\sum_i^N \boldsymbol{k}_i)$$

# Double DGLAP equation and sum rules

Can use LC wavefunction representation of dPDF + LC perturbation theory to demonstrate that the dPDF so defined satisfies the double DGLAP equation. Method similar to Burkardt, Ji, Yuan [hep-ph/0205272] and Harindranath, Kundu, Zhang [Phys Rev D59, 094013].

Moreover, can show explicitly from LC representation of dPDF that dPDFs obey the momentum and number sum rules shown earlier.

# Probing dPDFs

Is there any process in which the dPDFs are probed directly – i.e. in which they appear explicitly in the formula for the cross section?

The kind of process we are looking for is one in which two particles probe the proton's content, and those particles are uncorrelated in transverse space over length scales of the order of the proton radius.

If the two probe particles come from different nucleons of a large *A* nucleus whose thickness does not vary much over the diameter of the proton, then we expect the ***b*** distribution of the pair of probe partons to be roughly uniform.

→ cross section expression for two-nucleon contribution to proton-nucleus DPS contains dPDF rather than 2pGPD. This result appears in the pN DPS cross section formulae of Strikman and Treleani [Phys.Rev.Lett., 88:031801]:

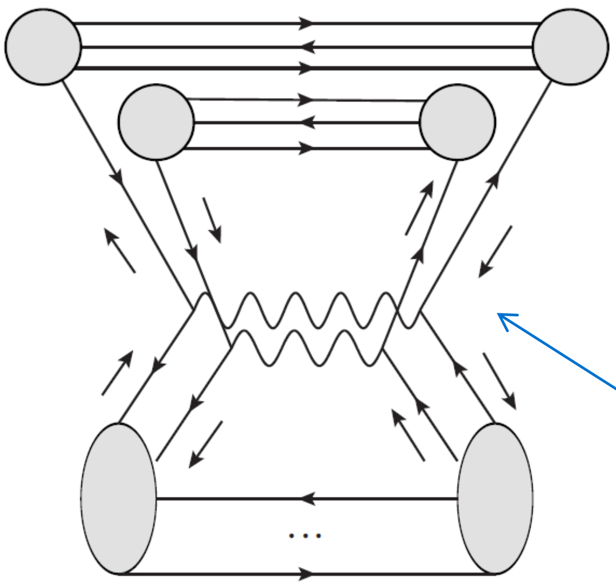
$$\sigma_2^D = \frac{1}{2}\int G_N(x_1,x_2)\hat{\sigma}(x_1,x_1')\hat{\sigma}(x_2,x_2')G_N(x_1')G_N(x_2')dx_1dx_1'dx_2dx_2'\int d^2BT^2(B)$$

$$G_N(x_1,x_2) = \int d^2b\Gamma_N(x_1,x_2;b)$$

Nuclear thickness function

# Interference contributions to 2 nucleon pN DPS

A consequence of having two independent probes is that interference diagrams play a less important role in two-nucleon pN DPS than they did in proton-proton DPS:




Partons returning to separate nucleons must have the same quantum numbers as they left with such that they can recombine with their respective spectators and reform original nucleons.

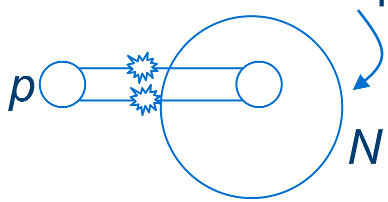
Angular momentum, colour and flavour conservation in the hard process ensures that the partons returning to the proton have the same quantum numbers (except when Ws are involved).

Will get a contribution to two-nucleon pN DPS associated with spin/colour correlations (not considered previously). Evolution of polarized dPDFs given by dDGLAP equation with unpolarized splitting functions replaced with polarized ones.

# Experimental measurement of two-nucleon pN DPS

Two-nucleon contribution to proton-heavy nucleus DPS is one way (perhaps the only practical way) to directly access the dPDF.

However, isolating this contribution experimentally is extremely challenging – there is a further contribution to pN DPS from the one-nucleon process, as well as SPS backgrounds.



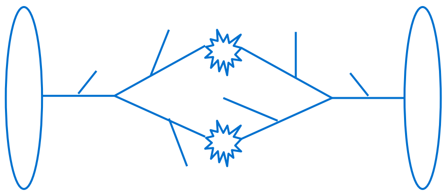

Potentially possible to separate out the contributions from the two DPS processes using their different $A$ dependencies – one-nucleon cross section varies with $A$ as $A^1$, whilst two-nucleon process goes like $A^{1.5}$ (Strikman, Treleani [Phys. Rev. Lett. 88:031801]).

This method requires one to measure proton-nucleus DPS for a number of different nuclei – unlikely that we will have such measurements in the near future.

# Use of GS09 dPDFs in pp DPS predictions

How ‘wrong’ is it to use the factorised assumption + GS09 dPDFs to calculate the DPS cross section?

At the very least, it is better than using naive factorised forms as it takes account of momentum and number constraints.



Certainly, in calculating the DPS cross section using GS09 you include erroneous contributions – in particular, you include ‘double parton splitting’ diagrams as part of the DPS rather than the SPS, and wrongly assign a log singularity to each splitting in the diagram.

Numerically this erroneous contribution makes up only about 1% of the DPS cross section – not too serious in practice?

Of course, in the dPDF framework we do not take account of interference contributions & contributions associated with spin/colour correlations.

# Summary

- To make predictions of p-p DPS cross sections, we need the 2pGPDs. There seems to be a breakdown on some level of the assumption that you can factorize a 2pGPD into transverse and longitudinal parts.
- The dPDFs that we have previously studied are the integrals of the 2pGPDs over transverse separation of the parton pair. These quantities obey the dDGLAP equation and our momentum and number sum rules.
- It is possible to directly probe the dPDFs via the two-nucleon contribution to p-A DPS, though it is difficult to measure this contribution in practice.
- GS09 dPDFs preferable to factorised forms even for pp DPS since they take account of momentum and number constraints.