UNIVERSITY OF CAMBRIDGE

Department of Physics
Cavendish Laboratory

Science & Technology Facilities Council

The GS09 Double Parton Distribution Functions

Jo Gaunt

Joint DAMTP and Cavendish Phenomenology Seminar, 27 May 2010

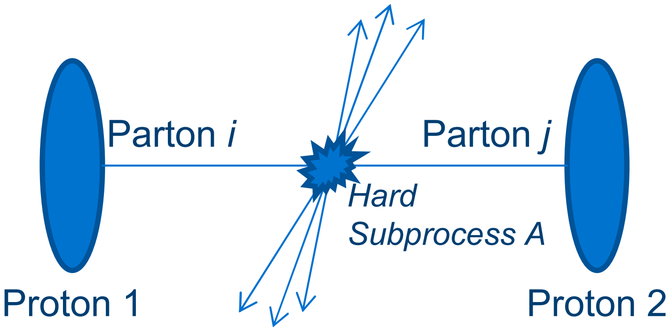
Work performed in collaboration with W.J. Stirling (arXiv:0910.4347), and C.H. Kom, A. Kulesza and W.J. Stirling (arXiv:1003.3953).

Outline

- Introduction
  - Single parton scattering, single PDFs and DGLAP
  - Double parton scattering, double PDFs and double DGLAP
- The number and momentum sum rules for the dPDFs.
- Development of the GS09 dPDFs:
  - Using the number and momentum sum rules to create improved inputs.
  - Numerical Implementation of dDGLAP equation.
- Difference between GS09 dPDFs and factorised forms previously used, in the context of same-sign WW DPS signal. Will we be able to measure this difference given the backgrounds?
- Summary

Single Parton Scattering

Single Parton Scattering (SPS):



$$\sigma_S^{(A)} = \sum_{i,j} \int D_h^i(x_1; Q_A) D_h^j(x_1'; Q_A) \hat{\sigma}_{ij}^A(x_1, x_1') \mathrm{d}x_1 \mathrm{d}x_1'$$

(Single) Parton Distribution Functions

Hard subprocess cross section

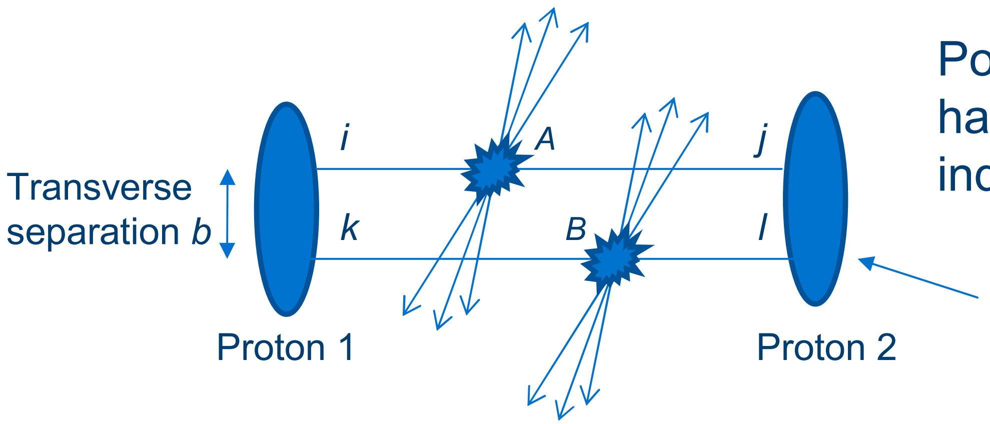
Whilst we cannot calculate the parton distributions using pQCD, we can determine precisely their variation with $t \equiv \ln(Q^2)$:

$$\frac{\partial D_h^i(x;t)}{\partial t} = \frac{\alpha_S(t)}{2\pi} \sum_j \int \frac{dz}{z} D_h^j(z;t) P_{j \to i}\left(\frac{x}{z}\right)$$

The DGLAP Equation

‘1-1 Splitting function’

Double Parton Scattering



Possibility exists to have more than one hard parton-parton scattering in an individual proton-proton collision

e.g. double parton scattering (DPS)

Can use factorisation theorem to deduce form of DPS cross section:

$$\sigma_D^{(A,B)} = \underset{\text{Symmetry factor}}{\frac{m}{2}} \sum_{i,j,k,l} \int \overbrace{\Gamma_h^{ik}(x_1, x_2, b; Q_A, Q_B)\Gamma_h^{jl}(x_1', x_2', b; Q_A, Q_B)}^{\text{Generalised double distributions}}$$

$$\times \underbrace{\hat{\sigma}_{ij}^{A}(x_1, x_1')\hat{\sigma}_{kl}^{B}(x_2, x_2')}_{\text{Parton level cross sections}} \mathrm{d}x_1\,\mathrm{d}x_1'\,\mathrm{d}x_2\,\mathrm{d}x_2'\,\mathrm{d}^2 b$$

Simplifying assumptions for DPS Cross Section

1. Take Γ to be a product of longitudinal and transverse pieces .

$$\Gamma_h^{ik}(x_1,x_2,b;Q_A,Q_B) = D_h^{ik}(x_1,x_2;Q_A,Q_B)\,F_k^i(b)$$

Double parton distribution functions (dPDFs)

Parton pair density in transverse space

2. Assume that *F* does not depend on parton indices – i.e. $F_k^i(b) = F(b)$

Then, if we define $\sigma_{eff} = \dfrac{1}{\int [F(b)]^2 d^2 b}$ we may write DPS cross section as:

$$\sigma_D^{(A,B)} = \frac{m}{2\sigma_{eff}} \sum_{i,j,k,l} \int D_h^{ik}(x_1,x_2;Q_A,Q_B) D_h^{jl}(x_1',x_2';Q_A,Q_B) \times \hat{\sigma}_{ij}^A(x_1,x_1') \hat{\sigma}_{kl}^B(x_2,x_2') \mathrm{d}x_1\,\mathrm{d}x_1'\mathrm{d}x_2\,\mathrm{d}x_2'$$

3. Neglect longitudinal correlations $D_h^{ij}(x_1,x_2;Q_A,Q_B) \approx D_h^i(x_1;Q_A) D_h^j(x_2;Q_B)$

Generally thought to be a good approximation at low x_i – though we have shown that this is not the case for all dPDFs (see later).

$$\Rightarrow \sigma_D^{(A,B)} \approx \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

Why do we normally ignore DPS?

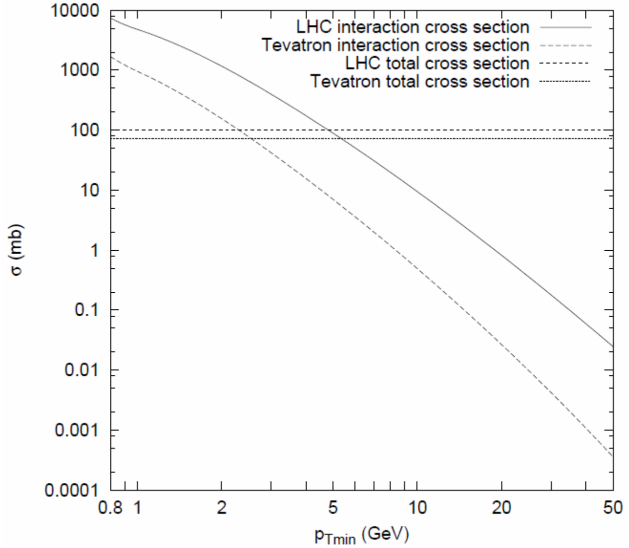
Probability that in a proton-proton collision at the LHC, a hard process will occur producing jets with $p_T > 20\ GeV \approx 10^{-2}$

$p_T > 50\ GeV \approx 10^{-4}$

$\Rightarrow$ hard parton-parton interactions are rare!

Can imagine that double parton interactions are suppressed by similar factors with respect to single interaction probability

$\Rightarrow$ DPS occurs rarely compared to SPS.



Skands and Sjöstrand, JHEP 03 053, 2004

Why then should we care about DPS at the LHC?

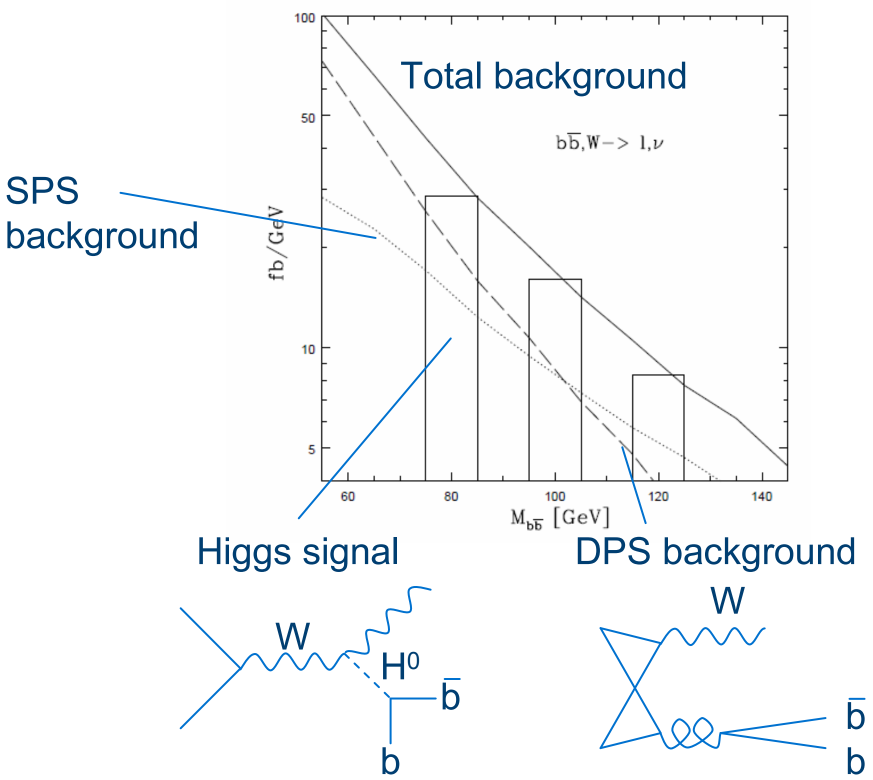
Consider crudest approximation for DPS cross section:

$$\sigma_D^{(A,B)} \approx \frac{m}{2}\frac{\sigma_S^{(A)}\sigma_S^{(B)}}{\sigma_{eff}}$$

⇒DPS cross sections go like the product of SPS ones!
⇒DPS cross sections **grow faster with energy than SPS** ***σ*.**

DPS processes...
• provide significant backgrounds to Higgs and new physics signals.
• reveal information about the structure of the proton.

DPS background to Higgs + W production (Del Fabbro and Treleani, hep-ph/9911358,1999):



Theoretical Ingredients of DPS Cross Section

$$\hat{\sigma}_{ij}^{A}(x_1, x_1')$$

Parton-level cross sections

•Known for essentially all processes of phenomenological interest

$$\sigma_{eff}$$

Effective Cross Section

•Non-perturbative.

•May actually depend on parton indices, hard scales and/or x_i due to partial violation of earlier assumptions 1 and 2 – unlikely to vary much with these variables however.

•Measured as $\sigma_S^{(A)}\sigma_S^{(B)}/\sigma_D^{(A,B)}$ by CDF and D0 collaborations, in an interaction and x_i region for which we are confident that the above ratio reliably gives σ_{eff}.

•We use the CDF value in our phenomenological studies (14.5mb) with the caveat that it should be re-measured at the LHC using appropriate benchmark processes.

$$D_h^{ik}(x_1, x_2; Q_A, Q_B)$$

Double parton distribution functions – **what do we know about these**?

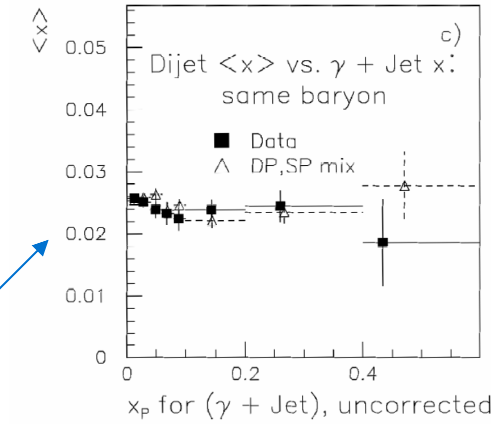
Experimental Studies

CDF and D0 studies of DPS contribution to γ+3 jets (A = 2*j*, B = γj).

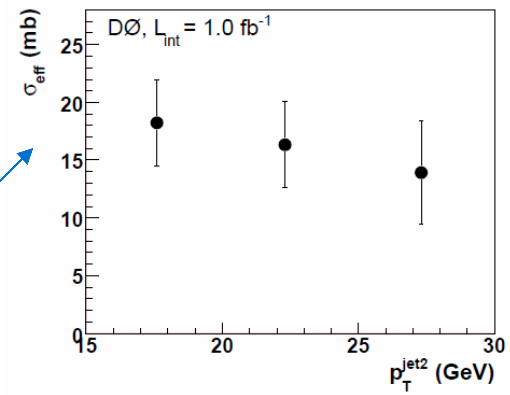
Majority of their events were produced by low *x* sea parton collisions.
For CDF sample, $0.01 < x < 0.40$ for partons producing γj, and $0.002 < x < 0.20$ for partons producing *jj*.

CDF investigated whether their data contained any evidence for *x* correlations between pairs of partons in the same proton. None found $\Rightarrow$ factorised approximation for dPDFs is good for sea quarks at low *x*.

D0 investigated variation of ratio $\sigma_S^{(A)}\sigma_S^{(B)}/\sigma_D^{(A,B)}$ with second largest jet p_T. Data consistent with no variation, although suggestion that ratio decreases with increase in p_T (effects of pQCD evolution on dPDFs?)



Phys. Rev. D56 3811–3832, 1997.



Phys. Rev. D81 052012, 2010.

Theoretical Work

Kirschner, Shelest, Snigirev, and Zinovjev have derived a ‘double DGLAP equation’ describing the change in the dPDFs with factorisation scale, for the special case of the dPDFs with $Q_A = Q_B = Q$.

$$\frac{dD_h^{j_1 j_2}(x_1, x_2; t)}{dt} = \frac{\alpha_s(t)}{2\pi}\left[\sum_{j_1'} \int_{x_1}^{1-x_2} \frac{dx_1'}{x_1'} D_h^{j_1' j_2}(x_1', x_2; t) P_{j_1' \to j_1}\left(\frac{x_1}{x_1'}\right)\right.$$
$$+ \sum_{j_2'} \int_{x_2}^{1-x_1} \frac{dx_2'}{x_2'} D_h^{j_1 j_2'}(x_1, x_2'; t) P_{j_2' \to j_2}\left(\frac{x_2}{x_2'}\right)$$
$$\left. + \sum_{j'} D_h^{j'}(x_1 + x_2; t) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}\left(\frac{x_1}{x_1 + x_2}\right)\right]$$

Usual 1→1 splitting functions

NEW!

‘1→2’ splitting function

Single PDF

(Kirschner, Phys.Lett.B84:266, 1979 and Shelest, Snigirev, and Zinovjev, Phys.Lett.B113:325,1982).

$1 \rightarrow 2$ splitting functions

$$\frac{\alpha_S(t)\Delta t}{2\pi} P_{i\rightarrow jk}(x)\delta x$$ = probability of an *i* parton with mtm 1 splitting to give a *j* parton with mtm *x* and a *k* parton with mtm (1-*x*) when scale is increased from *t* to *t*+Δ*t*

In the above, we have implicitly assumed that a single splitting can only give rise to two particles, such that *jk* carry all mtm of *i* $\Rightarrow$ $1\rightarrow 2$ splitting function with just one mtm argument only makes sense at LO.

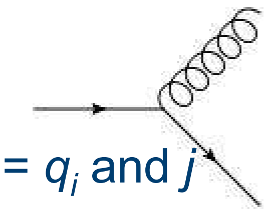
Higher order splitting function must have two mtm arguments:

$$P_{i\to jk}(x_1,x_2) = \delta(1-x_1-x_2)P^{(0)}_{i\to jk}(x_1) + \frac{\alpha_s}{2\pi}P^{(1)}_{i\to jk}(x_1,x_2) + \ldots$$

(& structure of last term in dDGLAP equation must be altered at NLO and above!)

$1\rightarrow 2$ trivially related to $1\rightarrow 1$ splitting functions in LO case.

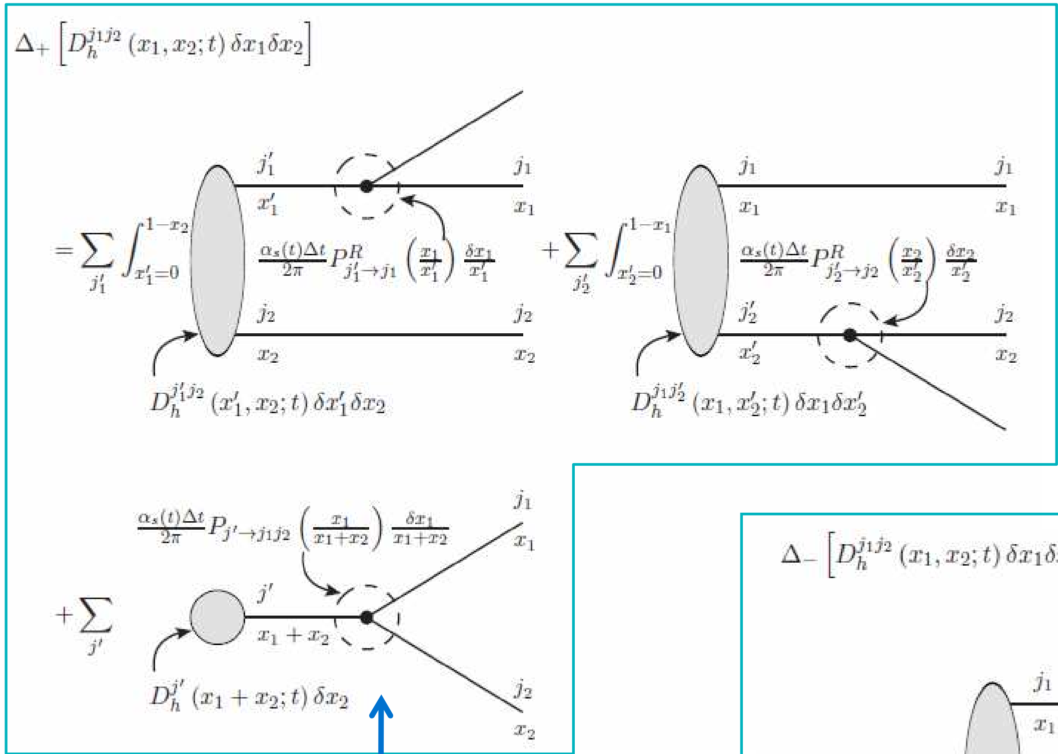
$$P_{i\to jk}(x) = \begin{cases} P^R_{i\to j}(x) & \text{if } k = \kappa(i,j) \\ 0 & \text{otherwise} \end{cases}$$



κ(*i*,*j*) = only choice of parton that can be combined with *i* & *j* to make a legitimate QCD 3-vertex.

e.g. if *i* = q_i and *j* = *g*, $\kappa = q_i$

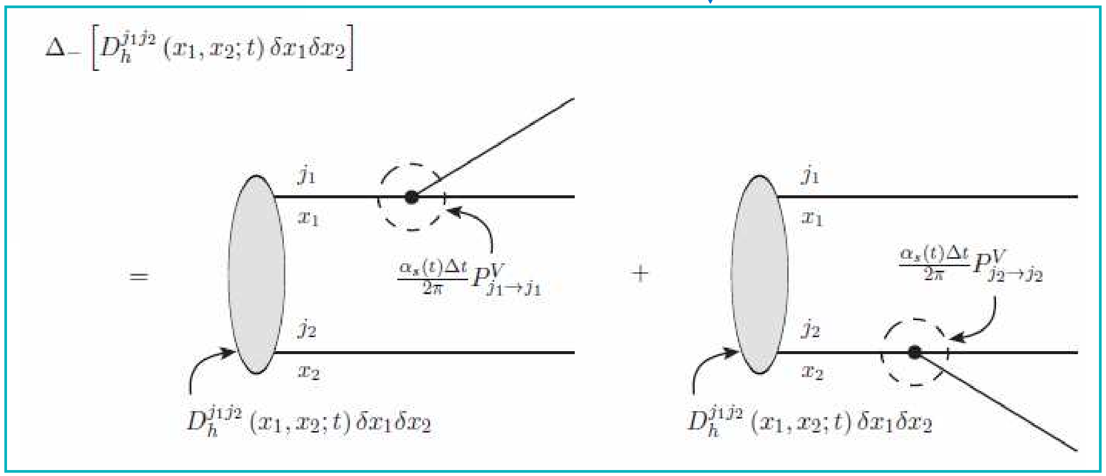
Pictorial representation of double DGLAP equation



Splitting processes acting to increase D^{ij} as the scale is increased from t $\to$ t +Δt.

Splitting processes acting to decrease D^{ij} as the scale is increased from t $\to$ t +Δt.

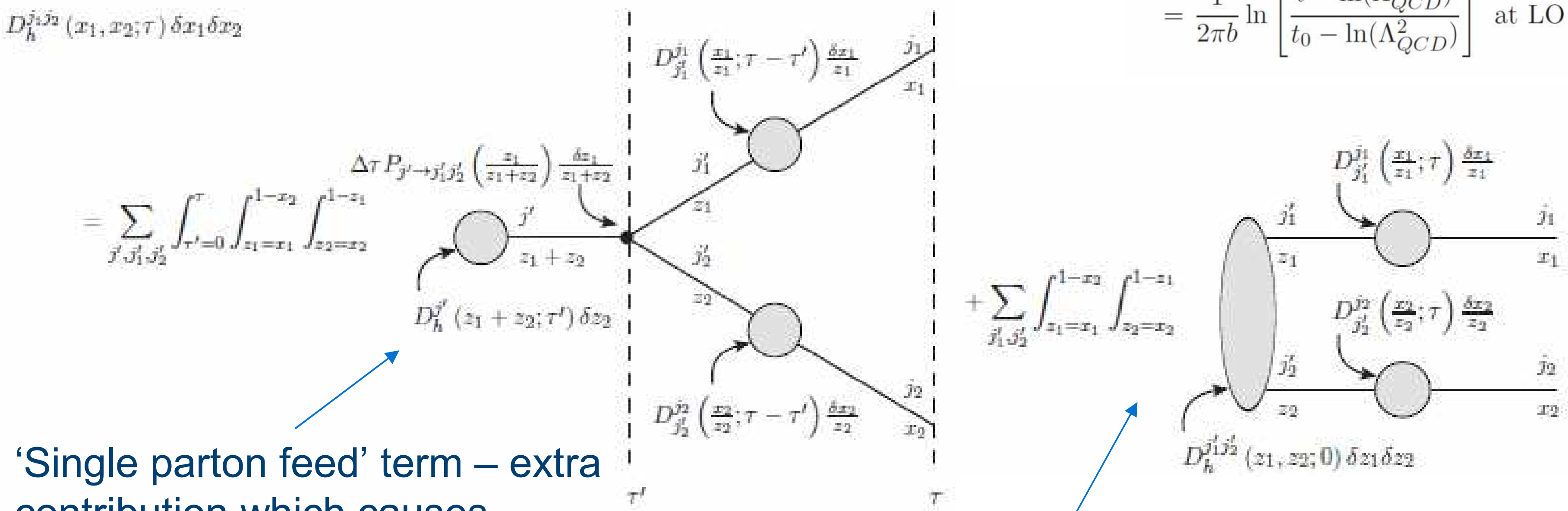
“single PDF feed”



Double DGLAP evolution as a branching process

$$\tau = \int_{t_0}^{t} dt' \frac{\alpha_s(t')}{2\pi}$$
$$= \frac{1}{2\pi b} \ln\left[\frac{t - \ln(\Lambda_{QCD}^2)}{t_0 - \ln(\Lambda_{QCD}^2)}\right] \text{ at LO}$$

Can solve dDGLAP equation to give, pictorially:

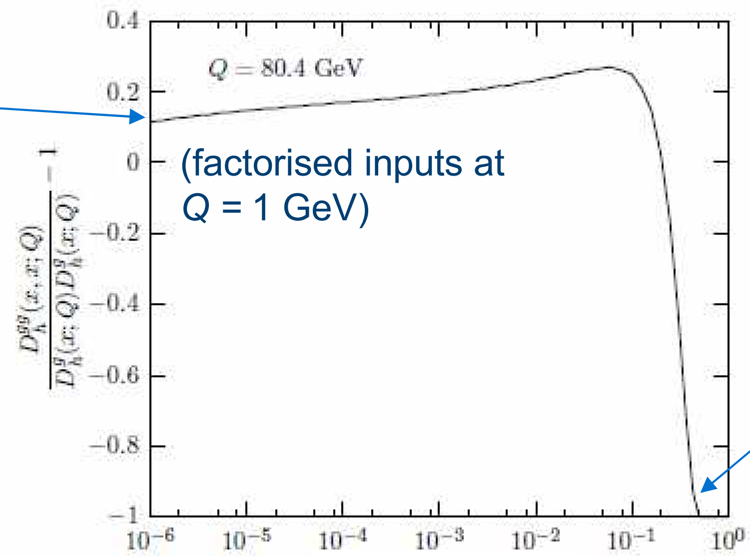


'Single parton feed' term – extra contribution which causes dPDFs to deviate from factorised forms, particularly at low *x*.

'Independent branching' term – largely preserves factorised forms, except at large *x* where mtm conserving integration limits lead to significant deviation from factorised forms.

Numerical demonstration that pQCD evolution causes dPDFs to deviate from factorised forms

~10% deviation at low *x* due to single parton feed contribution



$x_1 = x_2 = x$

pQCD evolution correctly takes account of momentum constraints – ratio of dPDF to factorised form quickly goes to zero on kinematic boundary.

JG and Stirling, 0910.4347, 2009

Input dPDFs

The most accurate approach to modelling the (equal scale) dPDFs is to use the double DGLAP equation along with some suitably chosen inputs at a low scale Q_0.

But what should the inputs look like? **Can we get any theoretical insight**?

First reaction - **NO**! A dPDF at any particular scale receives contributions from **non-perturbative physics**.

The dPDF Sum Rules

Actually – **YES**, we can! We have shown that the following equalities (**sum rule equalities**) are preserved by (LO) double DGLAP:

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 D_h^{j_1 j_2}(x_1, x_2; t) = (1 - x_2) D_h^{j_2}(x_2; t)$$

$$\int_0^{1-x_2} dx_1 D_h^{j_{1v} j_2}(x_1, x_2; t) = \begin{cases} N_{j_{1v}} D_h^{j_2}(x_2; t) & \text{when } j_2 \neq j_1 \text{ or } \bar{j}_1 \\ (N_{j_{1v}} - 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = j_1 \\ (N_{j_{1v}} + 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = \bar{j}_1 \end{cases}$$

These equalities are no more than the statements of conservation of momentum and quark number for the dPDFs, and are analogous to the following result in probability theory:

$$\int dx x^a f(X = x \cap Y = y) = E(X^a \mid Y = y) f(Y = y)$$

Integrated over all values *X* can take given *Y* = *y*.

Random variables

The dPDF Sum Rules

Sum rules satisfy non-trivial consistency checks:

e.g. $$\sum_{j_2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 x_2 D^{j_{1v} j_2}(x_1, x_2; t)$$ equates to the same number whether we use dPDF number sum rule + sPDF mtm sum rule or dPDF mtm sum rule + sPDF number sum rule

In general, we expect there to be a hierarchy of such relations, relating the integrals of *n* parton distributions to (*n*-1) parton distributions.

e.g. *n*PDF mtm sum rule: $$\sum_{j_1} dx_1 x_1 D_h^{j_1 j_2 \dots j_n}(x_1, x_2 \dots x_n) = \left(1 - \sum_{i=2}^{n} x_i\right) D_h^{j_2 \dots j_n}(x_2 \dots x_n)$$

The sum rules impose important constraints on the type of input dPDFs that are allowable...although non-trivial to implement them!

The GS09 dPDFs

JG and Stirling, 0910.4347, 2009

First set of publicly available LO equal-scale dPDFs (available from **HepForge***). Package includes grid of dPDF values spanning $10^{-6} < x_1 < 1$, $10^{-6} < x_2 < 1$, $1\ \mathrm{GeV}^2 < Q^2 < 10^9\ \mathrm{GeV}^2$ + interpolation code.

Two stages to the project leading to these dPDFs:

1. Construction of a set of low-scale inputs approximately satisfying the sum rules.

2. Development of a routine that is able to numerically integrate the dDGLAP equations and evolve the given inputs up to any other scale.

*http://projects.hepforge.org/gsdpdf/

The Input dPDFs

It is most convenient to work terms of the 'double evolution' parton flavour basis when trying to construct a set of dPDFs satisfying the sum rules (also when considering dPDF evolution).

In this basis, the parton indices *ij* of a dPDF are one out of the following rather than q_i, g, etc:

A dPDF with one of these indices will be involved in a number sum rule

$$j_v = q_j - \bar{q}_j \quad \text{Valence}$$

A dPDF with one of these indices will be involved in a mtm sum rule

$$\Sigma = \sum \left(q_j + \bar{q}_j \right) \quad \text{Singlet}$$

$$g \quad \text{Gluon}$$

'Tensor' combinations

$$q^+ = q_j + \bar{q}_j$$

$$T_3 = u^+ - d^+$$

$$T_8 = u^+ + d^+ - 2s^+$$

$$T_{15} = u^+ + d^+ + s^+ - 3c^+$$

$$T_{24} = u^+ + d^+ + s^+ + c^+ - 4b^+$$

$$T_{35} = u^+ + d^+ + s^+ + c^+ + b^+ - 5b^+$$

Must choose a set of sPDF inputs to which our dPDF inputs correspond – we use MSTW2008LO inputs (with some slight alterations – e.g. we take $s_v = 0$).

$\Rightarrow$ our input scale Q_0 = MSTW 2008 input scale = 1 GeV.

Can we base our dPDFs on factorised forms?

Wishing to make maximal use of the detailed information we have on sPDFs, and based on popular lore + experimental evidence from the Tevatron, we would like to use input dPDFs which are based around simple products of sPDFs, and become equal to such products in the low *x* limit. Can we do this & still satisfy the sum rules?

Yes – for all dPDFs **except** for the equal flavour valence-valence (EFVV) distributions. Consider number sum rule for these quantities:

$$\underbrace{\int_0^{1-x_2} dx_1 D_h^{j_v j_v}(x_1, x_2; t_0)} = \underbrace{N_{j_v} D_h^{j_v}(x_2; t_0) - D_h^{j+\bar{j}}(x_2; t_0)}$$

If dPDF is approximately factorised form for small x_1 and x_2, expect $\mathrm{LHS} \sim x_2^{-a_v} \sim x_2^{-0.5}$ for small x_2

Dominated by second term at low x_2 $\Rightarrow \mathrm{RHS} \sim -x_2^{-a_s} \sim -x_2^{-1}$ for small x_2

Take factorised forms as the basis for all other dPDFs, and come back to the problem of EFVV distributions later.

Key features desirable in our inputs

There are two key features that we would like to build in to our set of input dPDFs. These are the following:

1. The dPDFs should be suppressed below factorised values near the kinematical bound $x_1+x_2=1$ due to phase space considerations.
2. Terms should be added/subtracted from certain dPDFs to take account of number effects.

Consider requirement (1) first. In previous studies, factors such as $(1-x_1-x_2)$ or $(1-x_1-x_2)^2$ multiplying factorised forms have been advocated to take account of phase space effects.

Momentum sum rule tells us neither of these options are fully satisfactory. Consider mtm sum rules along the lines $x_1 = 0$ and $x_2 = 0$. Along these lines, all mtm sum rules are perfectly satisfied using factorised dPDFs, whilst dPDFs including a $(1 - x_1 - x_2)$ or $(1 - x_1 - x_2)^2$ factor violate the sum rules badly.

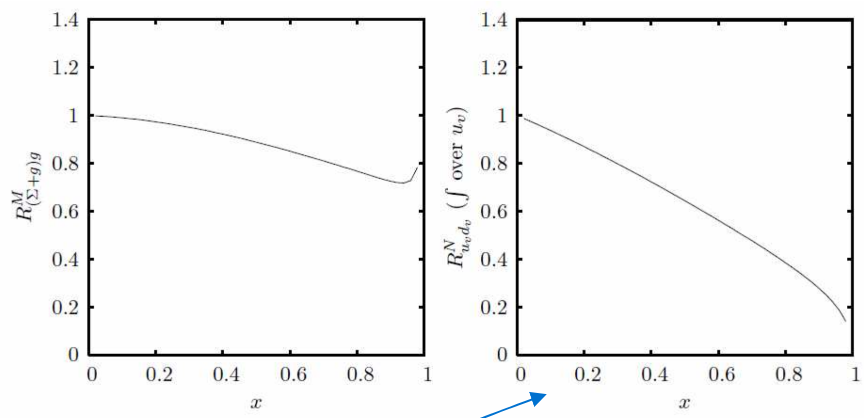
Taking account of phase space effects

We continue to follow tradition of previous papers in that we have attempted to apply a universal phase space factor to all of the dPDFs (has the advantage that it is guaranteed to produce positive 'double human flavour basis' dPDFs). Motivated by previous discussion, we first tried the following form for the phase space factor ρ:

$$\rho(x_1, x_2) = (1 - x_1 - x_2)^2 (1 - x_1)^{-2} (1 - x_2)^{-2}$$

Mtm sum rules reasonably well satisfied using this phase space factor.

However, number sum rules are not – this is true even for those dPDFs which are not affected by number effects, for whom the phase factor alone should be sufficient to ensure the dPDF satisfies the relevant number sum rule.



Taking account of phase space effects

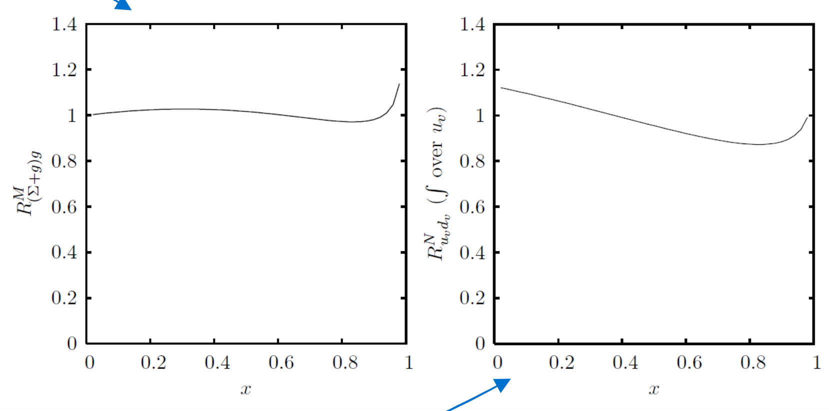
A slight adjustment to the original phase space factor solves this problem. Let ρ depend on the dPDF flavour indices *ij*, and define it to be the following:

$$\rho^{ij}(x_1, x_2) = (1 - x_1 - x_2)^2 (1 - x_1)^{-2-\alpha(j)} (1 - x_2)^{-2-\alpha(i)}$$

where $\alpha(i) = \begin{cases} 0 & \text{if } i \text{ is a sea parton} \\ 0.5 & \text{if } i \text{ is a valence parton} \end{cases}$

dPDFs are all still positive, and mtm sum rules still obeyed well (better!)

However, the extent to which the dPDFs not affected by number effects obey their number sum rules is significantly improved.



Taking account of number effects

Number effects can in principle have an impact on any dPDF for which the same parton type appears in both parton indices. However, since there are only a finite number of valence quarks in the proton (as opposed to infinite numbers of sea quarks and gluons), expect number effects relating to valence quarks to be most important.

e.g. $$u^+u^+ = (u_v + 2u_s)(u_v + 2u_s) = \underbrace{2u_s u_v + 2u_v u_s + 4u_s u_s}_{} + u_v u_v$$

Factorised form × phase space factor reasonable for these – extracting sea quark doesn't much affect chances of finding another sea quark or valence quark

However, approximate factorised form not adequate for this part – need to take account of the fact that finding a valence up quark halves the probability to find another.

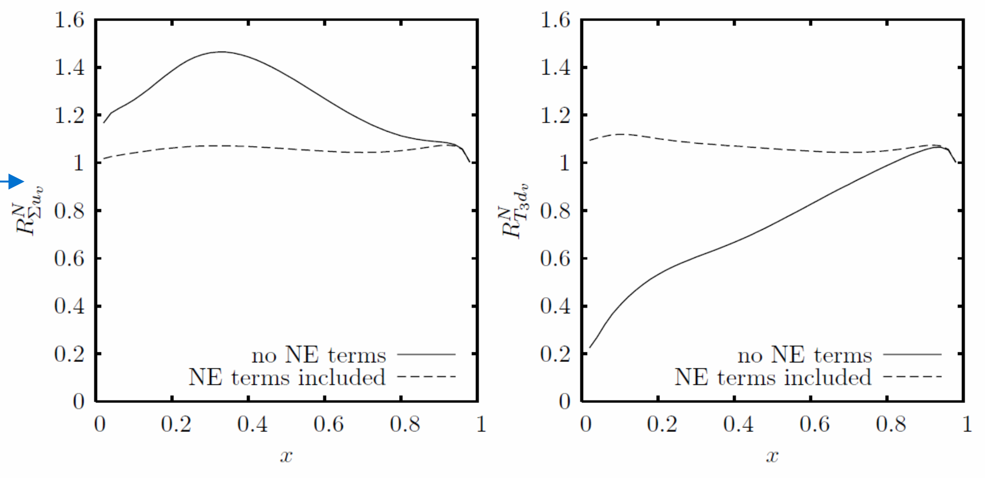
Taking account of number effects

In general, can take account of valence number effects in our dPDFs inputs by subtracting the following terms from our factorised forms × phase space factors:

$\frac{n}{2} D_h^{u_v}(x_1;t_0) D_h^{u_v}(x_2;t_0) \rho^{u_v u_v}(x_1,x_2)$ from dPDFs containing n times the $u_v u_v$ combination

$n D_h^{d_v}(x_1;t_0) D_h^{d_v}(x_2;t_0) \rho^{d_v d_v}(x_1,x_2)$ from dPDFs containing n times the $d_v d_v$ combination

Effect of including number effect terms on the extent to which the affected dPDFs satisfy their sum rules



Equal flavour valence-valence dPDFs

Need to construct suitable EFVV inputs for up, down, **and strange** flavours. Note that our $s_v s_v$ input cannot be zero even though we have taken s_v sPDF zero:

$$\int_0^{1-x_2} dx_1 D_h^{s_v s_v}(x_1, x_2; t_0) = -D_h^{s+}(x_2; t_0)$$

Intuitive explanation: $s_v s_v = ss - \bar{s}s - s\bar{s} + \bar{s}\bar{s}$ and we expect $\bar{s}s, s\bar{s}$ to be slightly larger than $ss, \bar{s}\bar{s}$ due to number effects.

Key idea utilised to construct EFVV inputs is the hypothesis that at some scale *t'* < t_0 only the three valence quarks in the proton may be resolved, and all sea distributions are zero (early GRV idea). At this scale, EFVV dPDFs are given by:

$$D_h^{j_v j_v}(x_1, x_2; t') = \frac{N_{j_v} - 1}{N_{j_v}} D_h^{j_v}(x_1; t') D_h^{j_v}(x_2; t') \tilde{\rho}^{j_v j_v}(x_1, x_2) \quad \text{(= 0 for } j = d \text{ or } s\text{)}$$

Valence-valence number effects.

Phase factor appropriate to scale *t'*

Equal flavour valence-valence dPDFs

In evolution from *t'* to t_0 , independent branching terms of dDGLAP equation will only serve to take initial form of an EFVV distribution into its equivalent at t_0. On the other hand, sPDF feed terms will result in an extra contribution appearing in each EFVV dPDF.

Only $-\bar{j}j-j\bar{j}$ component of an EFVV dPDF receives sPDF feed contributions during the evolution – these are of the form:

$$-2\frac{\alpha_s(t)}{2\pi}D_h^g(x_1+x_2;t)\frac{1}{x_1+x_2}\boxed{P_{qg}\left(\frac{x_1}{x_1+x_2}\right)}$$

approx constant

$\Rightarrow$total sPDF feed contribution to EFVV distributions is roughly speaking, just a function of x_1 + x_2:

$$D_h^{j_v j_v}(x_1,x_2;t_0)=\frac{N_{j_v}-1}{N_{j_v}}D_h^{j_v}(x_1;t_0)D_h^{j_v}(x_2;t_0)\rho^{j_v j_v}(x_1,x_2)-2g^{j\bar{j}}(x_1+x_2;t_0)$$

c.f. number sum rule for this dPDF:

$$\int_0^{1-x_2}dx_1 D_h^{j_v j_v}(x_1,x_2;t_0)=(N_{j_v}-1)D_h^{j_v}(x_2;t_0)-2D_h^{\bar{j}}(x_2;t_0)$$

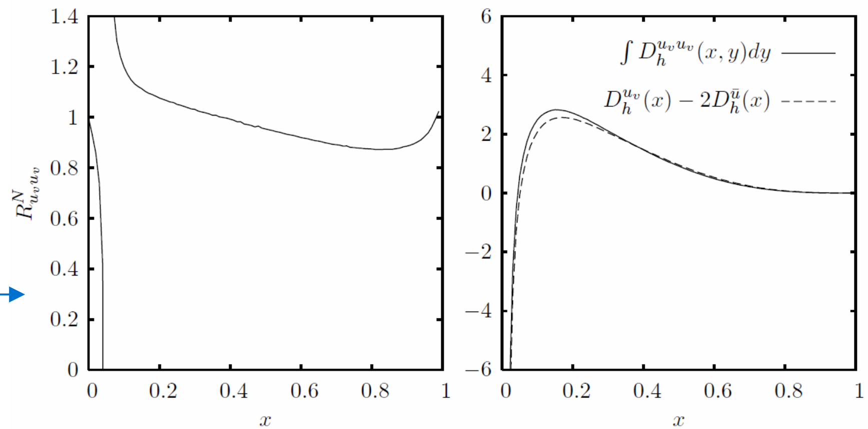
Equal flavour valence-valence dPDFs

$j\bar{j}$ correlation term must satisfy:

$$-2\int_0^{1-x_2} dx_1 g^{j\bar{j}}(x_1+x_2;t_0) = -2D_h^{\bar{j}}(x_2;t_0)$$

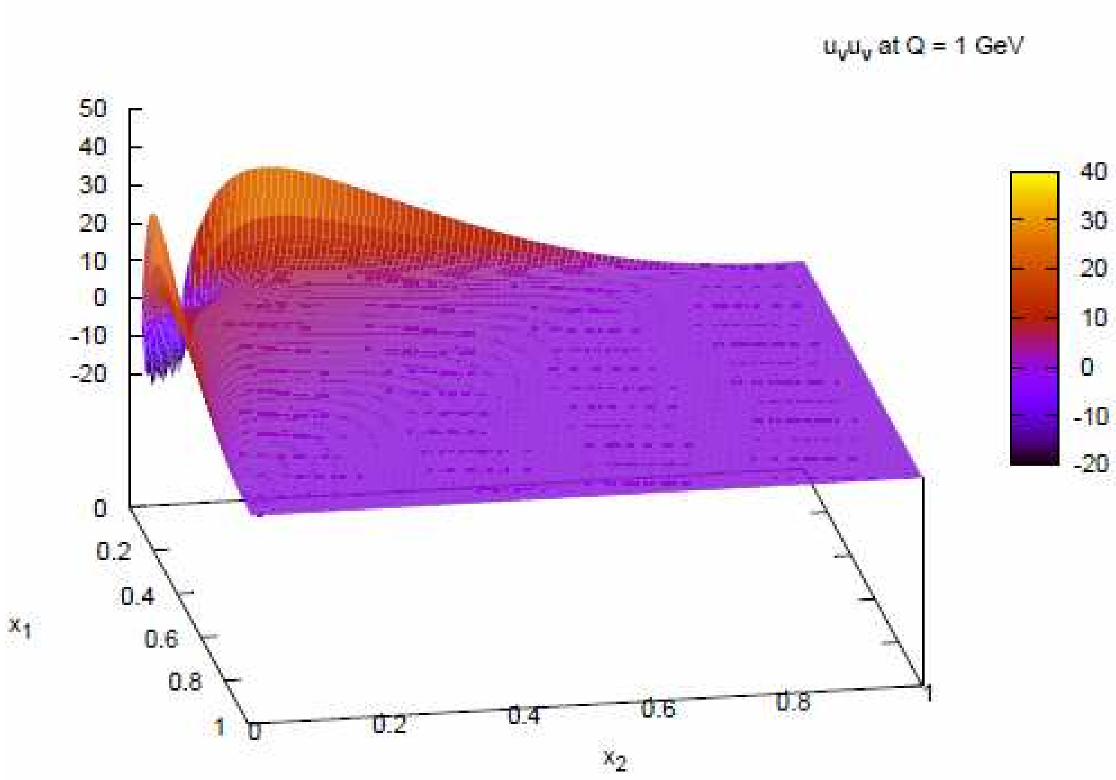
This is easy to solve, giving: $g^{j\bar{j}}(x;t_0) = -\frac{\partial D_h^{\bar{j}}(x;t_0)}{\partial x}$

This prescription gives $d_v d_v$ and $s_v s_v$ distributions which exactly satisfy their respective sum rules. Extent to which $u_v u_v$ distribution satisfies its sum rule:



Divergence in the sum rule ratio is caused by the integral curve slightly missing a zero in the sPDF quantity it should be equal to.

$u_v u_v$ input dPDF



Final adjustments

Unfortunately, using this scheme to construct the EFVV results in some negative double human flavour basis dPDFs – reason why we get some negative PDFs is that we've so far missed out an important term in our treatment of the j^+j^+ dPDFs.

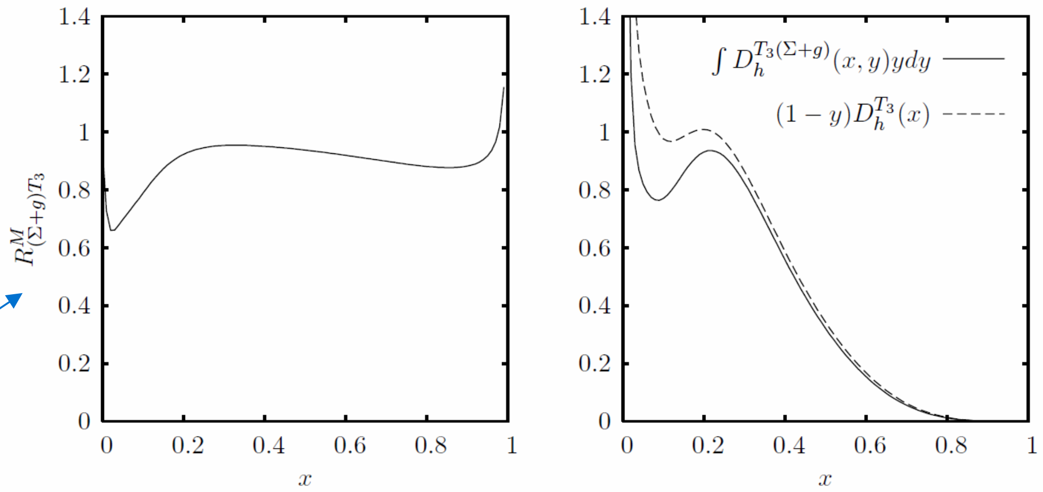
These distributions contain the same $\bar{j}j + j\bar{j}$ combination that is also found in the EFVV distributions, but here it appears with the opposite sign. For consistency, we should include a contribution to the j^+j^+ dPDFs equal to **plus** $2g^{j\bar{j}}(x_1 + x_2; t_0)$

With this adjustment all dPDFs positive and we see little adverse effect on the extent to which the sum rules are satisfied.

Summary of extent to which our inputs satisfy sum rules

In human flavour basis, **all sum rule ratios within 25% of 1 for *x* < 0.8**

In double evolution basis, similar story, barring trivial divergences. Only exception is $(\Sigma+g)T_3$ mtm sum rule:



Numerical Integration of dDGLAP

Numerical integration of dDGLAP equation performed directly in *x* space using a grid evenly spaced in the variable $u \equiv \ln(x/(1-x))$ in both *x* directions, and evenly spaced in *t*.

Combination of open + closed Newton-Cotes rules used to evaluate dDGLAP integrals

Runge-Kutta fourth-order method used for stepwise evolution in *t*.

Option to perform evolution under FFNS with $n_f = 3,4,5$, or 6, or ZM-VFNS with n_f potentially varying between 3 & 6.

$$I(y) = \int_x^{1-y} \frac{dz}{z} D(z,y) P\left(\frac{x}{z}\right)$$

$$P(x) = A(x) + K\delta(1-x) + R(x)[S(x)]_+$$

Approximate using grid

$$I(y) \approx \sum_{j=i}^{k} P_{ijk} D(x_j, y),$$

Jacobian

$$J(x) \equiv dx/du = x(1-x)$$

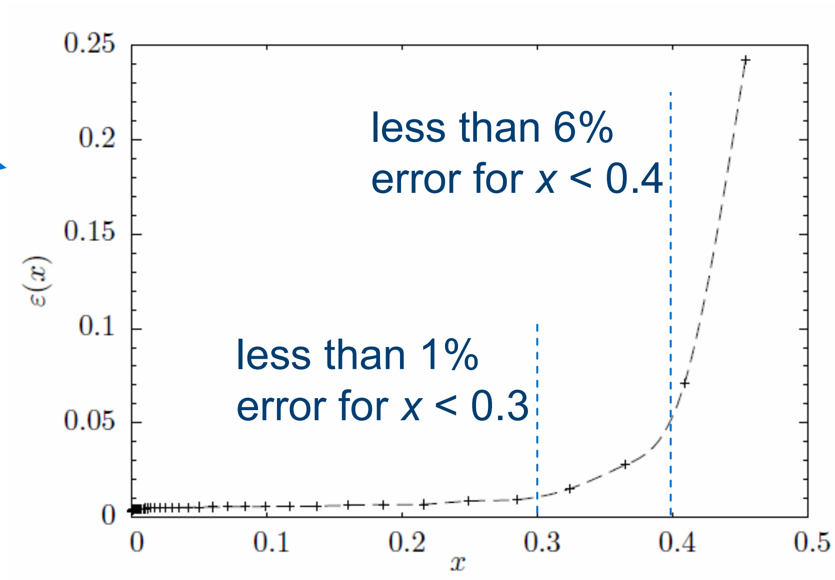
$$P_{ijk} = \begin{cases} \left[A\left(\frac{z_i}{z_j}\right) + R\left(\frac{z_i}{z_j}\right) S\left(\frac{z_i}{z_j}\right)\right] w_{ijk} \frac{J(z_j)}{z_j} \Delta u & \text{if } i < j \le k \\ K - R(1) \int_0^{x/(1-y)} dz S(z) & \\ - \sum_{j=i+1}^{k} S\left(\frac{z_i}{z_j}\right) \frac{z_i}{z_j} R(1) w_{ijk} \frac{J(z_j)}{z_j} \Delta u & \text{if } j = i,\ i < k \\ 0 & \text{otherwise.} \end{cases}$$

Newton-Cotes type integration weights

Numerical Integration of dDGLAP

Estimate of error introduced by numerical integration in an evolution from Q_0 = 1 GeV to Q_f = 100 GeV using 150 points in each x direction, and 10 in the t.

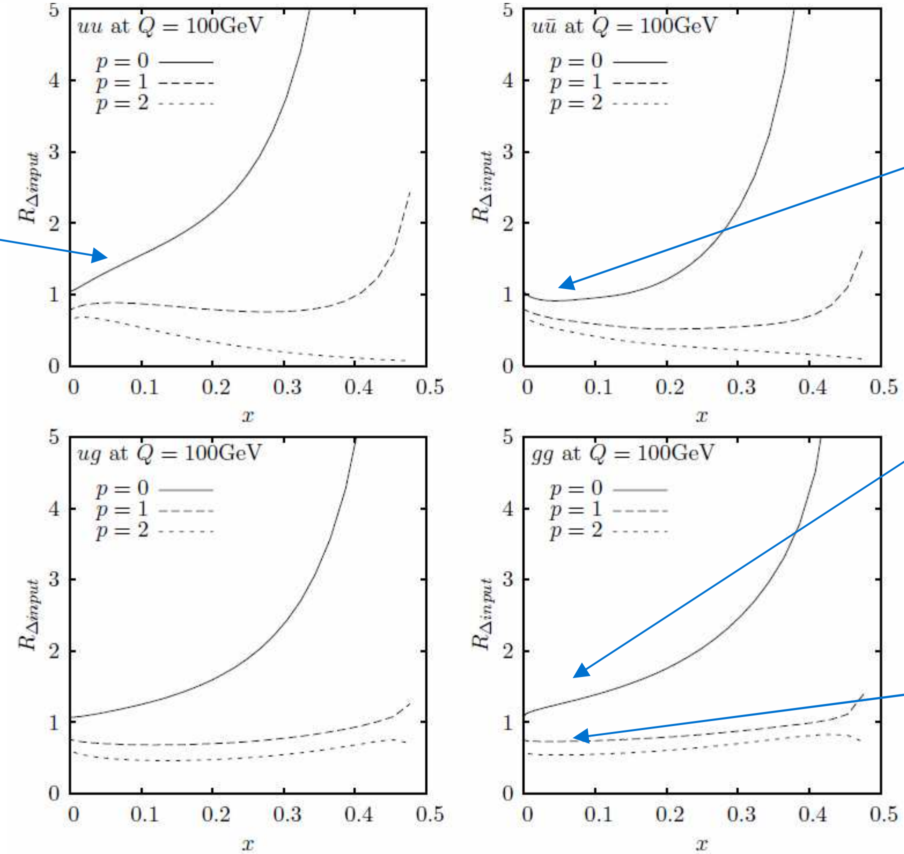
To produce publicly available grids, 600 points in each x direction and 120 in the t were actually used.



Effect of change in inputs on *Q = 100* GeV dPDFs

$$R^{ij}_{\Delta input}(x;Q) \equiv \frac{D_h^{ij}(x,x;Q)\,\big|_{\text{input } D_h^{ij}(x_1,x_2;Q_0)=D_h^i(x_1;Q_0)D_h^j(x_2;Q_0)(1-x_1-x_2)^p}}{D_h^{ij}(x,x;Q)\,\big|_{\text{input } D_h^{ij}(x_1,x_2;Q_0)=\text{our improved inputs}}}$$

Valence number effect subtractions from GS09 dPDF



Extra $j\bar{j}$ correlation term in GS09 $u\bar{u}$ input

p = 0 factorised input too large at large *x* – excess filters down to lower *x* values during evolution. Conversely, *p* = 1,2 inputs too small at large x – again this filters down during evolution.

Effect of sPDF feed on Q = *100* GeV dPDFs

$$R^{ij}_{\text{no feed}}(x;Q) \equiv \frac{D^{ij}_h(x,x;Q)\,\big|_{\text{our improved inputs, no sPDF feed}}}{D^{ij}_h(x,x;Q)\,\big|_{\text{our improved inputs}}}$$

sPDF feed makes approximately universal contribution to all dPDFs at small *x* (~10%)

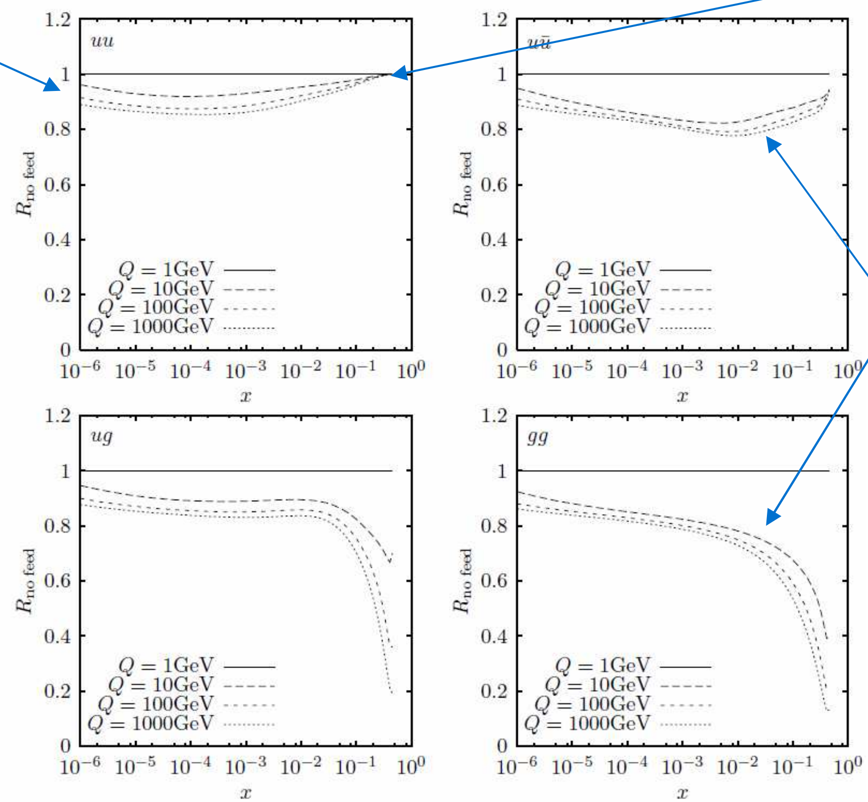
This is because dominant sPDF feed contribution to all dPDFs at low x comes from:

$$g \xrightarrow{1\to2\text{ splitting}} gg$$

independent branching

$$gg \rightarrow ij$$

No direct sPDF feed

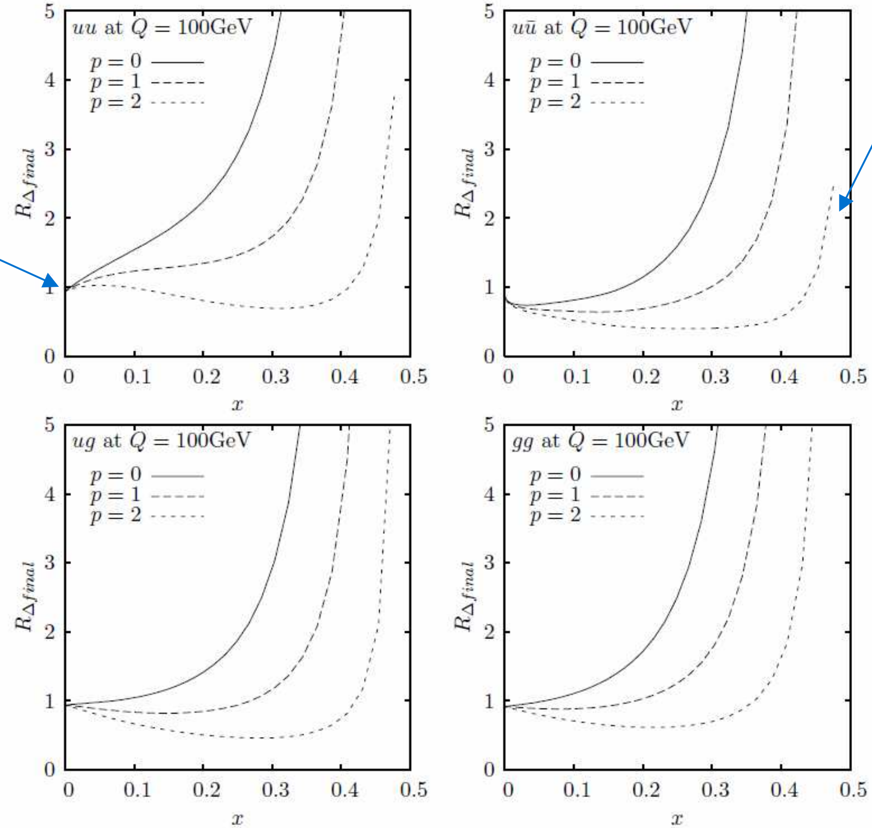


Gluon type evolution pulls PDFs to lower x values a lot more strongly than quark type evolution → dPDFs with gluon indices are more strongly affected at large x by removal of sPDF feed.

Effect of inputs + pQCD evolution on $Q = 100$ GeV dPDFs

$$R^{ij}_{\Delta final}(x_1, x_2; Q) \equiv \frac{D^i_h(x_1;Q) D^j_h(x_2;Q)(1-x_1-x_2)^p}{D^{ij}_h(x_1,x_2;Q)\,|_{\text{our improved inputs}}}$$

All ratios are about 10% below 1 at small x (lack of sPDF feed in factorised forms)



At Q = 100 GeV, even a $(1 - x_1 - x_2)^2$ suppression factor multiplying factorised forms represents an underestimate in the large x falloff of the dPDFs.

Comparison of GS09 with factorised dPDFs

JG, Kom, Kulesza, Stirling, 1003.3953, 2010

Comparison in the context of a particular process – **equal sign W pair production.**

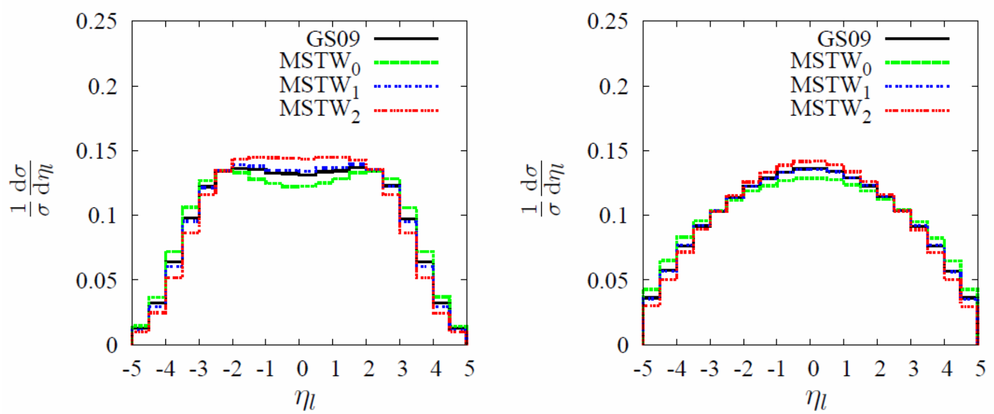
MSTW_n = product of MSTW sPDFs × $(1 - x_1 - x_2)^n$

Cross sections similar. MSTW_1 and MSTW_2 sets give smaller cross sections due to $(1 - x_1 - x_2)^{1,2}$ suppression of dPDFs.

$\sqrt{s} = 14$ TeV

	σ_{GS09}	σ_{MSTW_0}	σ_{MSTW_1}	σ_{MSTW_2}
W^+W^-	0.546	0.496	0.409	0.348
W^+W^+	0.321	0.338	0.269	0.223
W^-W^-	0.182	0.182	0.156	0.136
	R			
	0.784	1.00	1.00	1.00

Pseudorapidity distribution of leptons is similar with GS09 and MSTW_n (MSTW_1 gives best match).



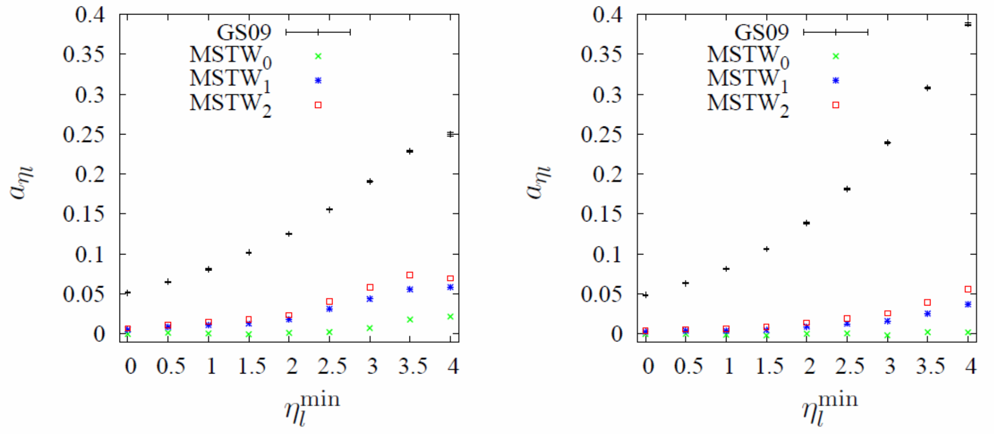
(a) Positively charged leptons (b) Negatively charged leptons

Lepton Pseudorapidity Asymmetry

However, it is possible to construct physical observables that are sensitive to the correlations inherent in GS09:

opposite hemisphere same hemisphere

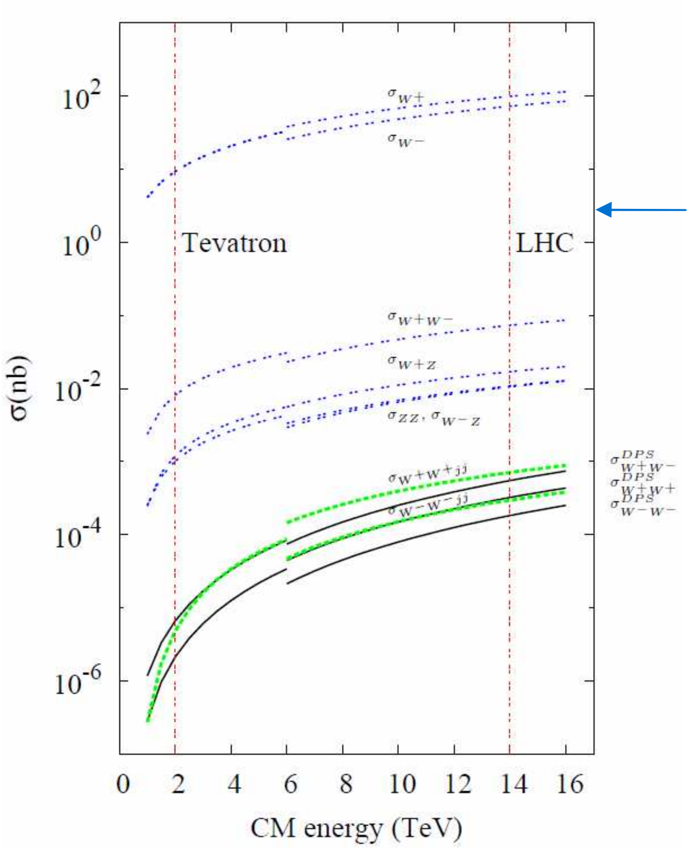
$$a_{\eta_l} = \frac{\sigma(\eta_{l_1} \times \eta_{l_2} < 0) - \sigma(\eta_{l_1} \times \eta_{l_2} > 0)}{\sigma(\eta_{l_1} \times \eta_{l_2} < 0) + \sigma(\eta_{l_1} \times \eta_{l_2} > 0)}$$



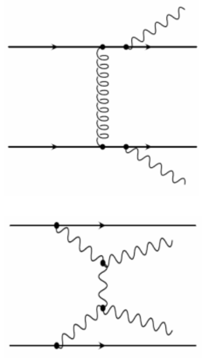
(a) Positively charged leptons (b) Negatively charged leptons

$a_{\eta l}$ larger for GS09 due to number effect subtractions, especially for large η_l^{min} (i.e. large x, where number effect subtractions have the largest impact).

Possibility of observing SSWW DPS at LHC

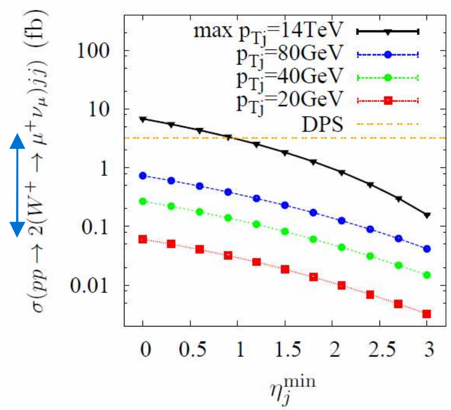


SPS same-sign WW production is forbidden at order $\alpha_W^2 \Rightarrow \sigma$ for this process is comparable to DPS σ, and always involves 2j.



This SPS background can be efficiently removed via a jet veto.

~2 orders of magnitude!



Other backgrounds to SSWW DPS

There are other SPS processes that can mimic the DPS same sign lepton signal:

•Heavy flavour

$$gg \to b\bar{b} \to B\bar{B} + \ldots\,, \qquad t \to W^+ b \to l^+ \nu b,$$
$$B \to l^+ \nu X, \qquad \bar{t} \to W^- \bar{b} \to q\bar{q}' l^+ \nu c.$$
$$\bar{B}^0 \to B^0 \to l^+ \nu \tilde{X},$$

•Electroweak gauge boson pair $Z(\gamma^*)Z(\gamma^*) \to l^+(l^-)l^+(l^-)$ $W^+Z(\gamma^*) \to l^+\nu l^+(l^-)$

(If these are not detected)

Thus this channel is not as 'clean' with regards to DPS as had been previously thought – carefully chosen cuts required to enhance S/B sufficiently.

	$\sigma_{\mu^+\mu^+}$ (fb)	$\sigma_{\mu^-\mu^-}$ (fb)
$W^\pm W^\pm$(DPS)	0.82	0.46
$W^\pm Z(\gamma^*)$	5.1	3.6
$Z(\gamma^*)Z(\gamma^*)$	0.84	0.67
$b\bar{b}$ ($p_T^b \geq 20$ GeV)	0.43	0.43

cuts

$$\sigma_{b\bar{b}}(p_T \geq 20\ \text{GeV}) = 5.15\ \mu\text{b}$$

Future Work

Extend treatment to NLO!

- Need to compute $1 \rightarrow 2$ splitting functions at NLO (trivial at LO).
- Will need NLO coefficient functions for certain benchmark processes (e.g. equal sign WW production).

Summary

- Important to understand DPS – will produce significant backgrounds and interesting signals at the LHC.
- For DPS predictions, require dPDFs. A 'double DGLAP' equation exists dictating the evolution of the equal-scale dPDFs, and we have derived the number and momentum sum rules for these quantities.
- We have produced the first publicly available set of LO equal-scale dDPFs. Sum rules used to guide construction of inputs at Q_0 = 1 GeV, and double DGLAP equation used to obtain dPDF values at other scales.
- Number and momentum correlations in GS09 dPDFs affect the signatures of DPS processes – but may be difficult to see this at LHC due to SPS background.

UNIVERSITY OF CAMBRIDGE

Backup Slides

Cuts to reduce ESWW DPS background

Modelling detector acceptance

The basic cuts are as follows:

1. Both leptons in the like sign lepton pair must have pseudorapidity $|\eta| < 2.5$.
2. Both leptons are required to be isolated: $E^l_{\rm ISO} \le E^{\rm min}_{\rm ISO} = 10$ GeV, where $E^l_{\rm ISO}$ is the hadronic transverse energy in a cone of $R = 0.4$ surrounding each of the like-sign leptons.
3. The transverse momenta of both leptons, p^l_T, must satisfy $20 \le p^l_T \le 60$ GeV.
4. An event is rejected whenever a third, opposite-signed, lepton is identified. A lepton is assumed to be identified with 100% efficiency when $p^l_T \ge p^{\rm id}_T$ and $|\eta| < \eta^{\rm id}$, where $p^{\rm id}_T = 10$ GeV and $\eta^{\rm id} = 2.5$.
5. The missing transverse energy $\not{E}_T$ of an event must satisfy $\not{E}_T \ge 20$ GeV.
6. Reject an event if a charged (lepton) track with $p^{\rm id}_T \ge p_T \ge 1$ GeV forms an invariant mass < 1 GeV with one of the same-sign leptons.

$b\bar{b}$

$W^+Z(\gamma^*), Z(\gamma^*)Z(\gamma^*)$

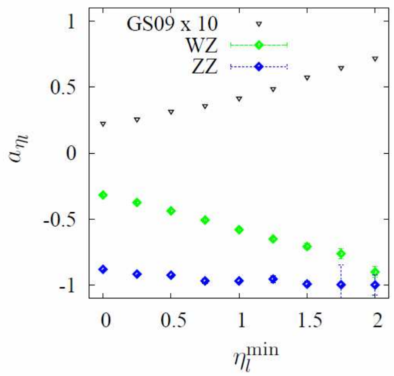
$b\bar{b}$

+ jet veto (p_T > 20 GeV) ← $W^+W^-jj, t\bar{t}$

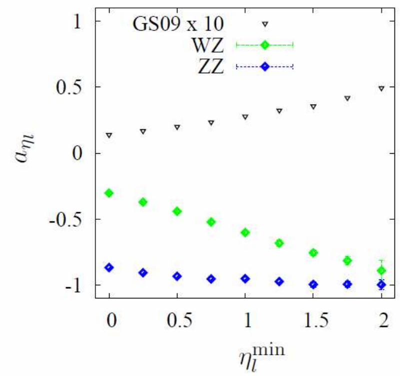
Further ways of discriminating signal from background

In DPS process, leptons more likely to be in opposite hemispheres (valence number effects), whilst SPS backgrounds tend to produce both leptons in the same hemisphere (small $\Delta\eta$ of final states corresponds to small partonic centre-of-mass energy)

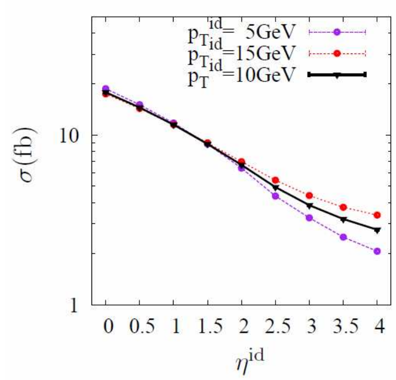
DPS processes give rise to a larger ratio of ++ to – leptons than background processes.



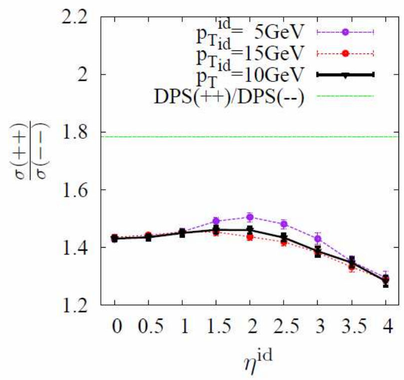
(a) +ve charged SSL events



(b) -ve charged SSL events



(a) $\sigma(W^+Z(\gamma^*))$ (fb)



(b) $\sigma(W^+Z(\gamma^*))/\sigma(W^-Z(\gamma^*))$