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Double Parton Scattering

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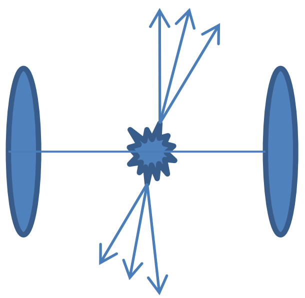
Liverpool HEP Theory Seminar, 23rd May 2012

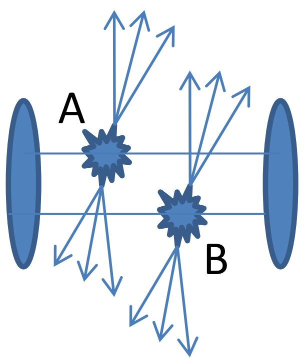
Outline

- Introduction to double parton scattering (DPS)
  - Description of the DPS cross section using 2pGPDs.
  - Why should we care about DPS at the LHC?
  - Experimental measurements of DPS.
- Description of a long-established framework for calculating the DPS cross section – the 'dPDF framework'. Summary of recent work I did in collaboration with J. Stirling in which we pointed out that this framework appears to be unsatisfactory.
- Discussion of different types of diagram that can contribute to DPS. Suggestion for total cross section for DPS.
- Interference and correlated parton contributions to DPS.
- Summary

Double Parton Scattering



In the standard theoretical framework for p-p scattering, we assume that a given collection of hard outgoing particles can only have been produced from the collision of two partons, one from either proton. This is single parton scattering (SPS).



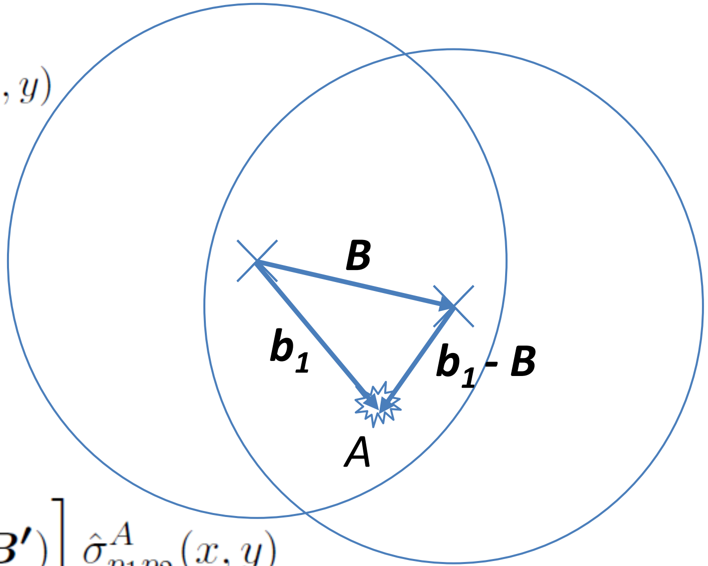
However, for certain final states, the possibility exists that the final state could have been produced as the result of two independent hard scatterings (**double parton scattering, or DPS**).

SPS cross section

Write in terms of impact parameter dependent PDFs:

$$\sigma^S_{(A)} = \sum_{p_1 p_2} \int dx dy d^2\boldsymbol{b}_1 d^2\boldsymbol{B} D_h^{p_1}(x, \boldsymbol{b_1}) D_h^{p_1}(y, \boldsymbol{b_1} - \boldsymbol{B}) \hat{\sigma}^A_{p_1 p_2}(x, y)$$

Corresponding picture in impact parameter space:



Can make a change of transverse variables in this formula and write:

$$\sigma^S_{(A)} = \sum_{p_1 p_2} \int dx dy \left[\int d^2\boldsymbol{b}_1 D_h^{p_1}(x, \boldsymbol{b_1}) \right] \left[\int d^2\boldsymbol{B'} D_h^{p_1}(y, \boldsymbol{B'}) \right] \hat{\sigma}^A_{p_1 p_2}(x, y)$$

$$= \sum_{p_1 p_2} \int dx dy D_h^{p_1}(x) D_h^{p_1}(y) \hat{\sigma}^A_{p_1 p_2}(x, y)$$

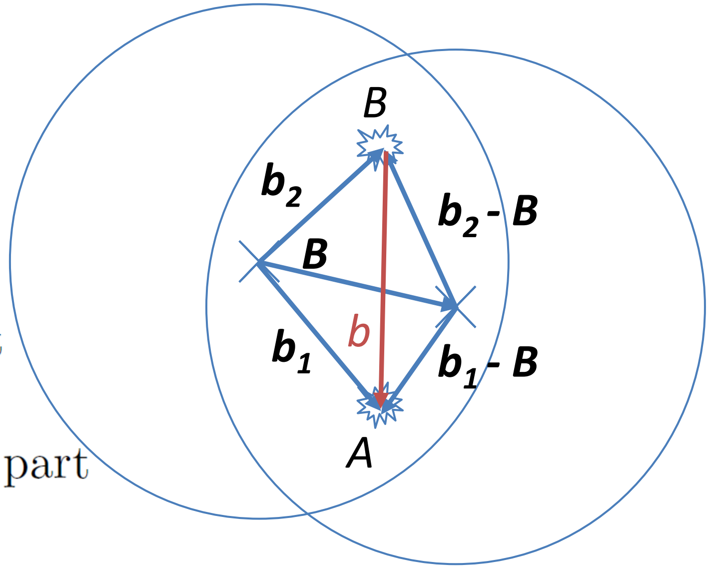
→ enough transverse momentum integrations to write cross section in terms of integrated sPDFs

DPS cross section

Assuming only factorisation of hard parts:

$$\sigma^D_{A,B} = \int d^2\boldsymbol{b_1} d^2\boldsymbol{b_2} d^2\boldsymbol{B} D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_2) D_h^{p_3p_4}(x_1',x_2';\boldsymbol{b}_1-\boldsymbol{B},\boldsymbol{b}_2-\boldsymbol{B}) \times \text{hard part}$$

Geometrical picture:



Changing variables:

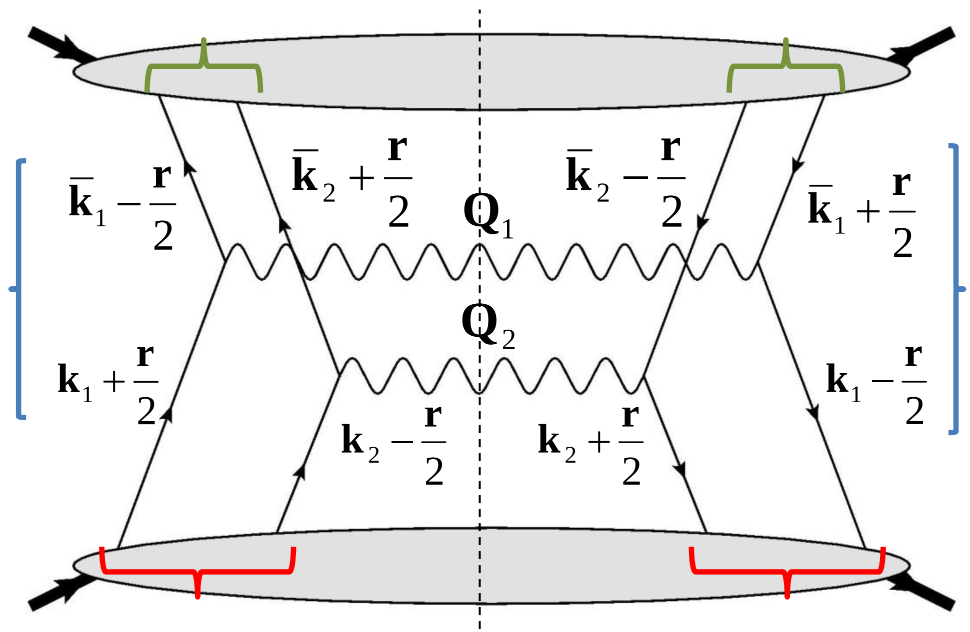
$$\sigma^D_{A,B} = \int d^2\boldsymbol{b} \int d^2\boldsymbol{b_1} D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_1-\boldsymbol{b})$$
$$\times \int d^2\boldsymbol{B'} D_h^{p_3p_4}(x_1',x_2';\boldsymbol{B'},\boldsymbol{B'}-\boldsymbol{b}) \times \text{hard part}$$
$$= \int d^2\boldsymbol{b} D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}) D_h^{p_3p_4}(x_1',x_2';\boldsymbol{b}) \times \text{hard part}$$

where: $D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}) = \int d^2\boldsymbol{b}_1 d^2\boldsymbol{b}_2 D_h^{p_1p_2}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_2)\delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_1+\boldsymbol{b}_2)$ **'2pGPD'**

Not enough transverse momentum integrations compared to constraints to write the DPS cross section in terms of fully integrated PDFs. Related to the fact that parton pairs from both protons must be separated by the same amount for double interaction.

DPS – transverse momentum space picture

Key point here – can have contributions from diagrams that are non-diagonal in transverse momentum space. Given the constraint that all initial & final state particles must have same momentum in amplitude and conjugate, most general form of such diagrams is →



r = momentum imbalance of a parton line between amplitude and conjugate

$$\sigma = \int \frac{d^2\mathbf{r}}{(2\pi)^2} \int \frac{d^2\mathbf{k}_1}{(2\pi)^2} \frac{d^2\mathbf{k}_2}{(2\pi)^2} D_h^{p_1 p_2}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{r}) \int \frac{d^2\bar{\mathbf{k}}_1}{(2\pi)^2} \frac{d^2\bar{\mathbf{k}}_2}{(2\pi)^2} D_h^{p_3 p_4}(x_1, x_2, \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2, -\mathbf{r})$$

$$= \int \frac{d^2\mathbf{r}}{(2\pi)^2} D_h^{p_1 p_2}(x_1, x_2, \mathbf{r}) D_h^{p_3 p_4}(x_1, x_2, -\mathbf{r})$$

This is actually the Fourier transform of **b**-space 2pGPD wrt **b**!

Simplifying assumptions for DPS Cross Section

1. Take Γ to be a product of longitudinal and transverse pieces .

$$\Gamma_h^{ik}\left(x_1,x_2,b;Q_A,Q_B\right)=D_h^{ik}\left(x_1,x_2;Q_A,Q_B\right)F_k^i\left(b\right)$$

Double parton distribution functions (dPDFs)

Parton pair density in transverse space

2. Assume that *F* does not depend on parton indices – i.e. $F_k^i\left(b\right)=F\left(b\right)$

   Then, if we define $\sigma_{eff}=\dfrac{1}{\int\left[F(b)\right]^2 d^2b}$ we may write DPS cross section as:

$$\sigma_D^{(A,B)}=\frac{m}{2\sigma_{eff}}\sum_{i,j,k,l}\int dx_i dx_i' D_h^{ik}\left(x_1,x_2;Q_A,Q_B\right)D_h^{jl}\left(x_1',x_2';Q_A,Q_B\right)\hat{\sigma}_{ij}^{A}\left(x_1,x_1'\right)\hat{\sigma}_{kl}^{B}\left(x_2,x_2'\right)$$

If e.g. $F\left(b\right)=\dfrac{1}{2\pi R^2}\exp\left(-\dfrac{b^2}{2R^2}\right)$ then $\sigma_{eff}=\dfrac{1}{4\pi R^2}$

3. Neglect longitudinal correlations $D_h^{ij}\left(x_1,x_2;Q_A,Q_B\right)\approx D_h^{i}\left(x_1;Q_A\right)D_h^{j}\left(x_2;Q_B\right)$

$$\Rightarrow \sigma_D^{(A,B)}\approx\frac{m}{2}\frac{\sigma_S^{(A)}\sigma_S^{(B)}}{\sigma_{eff}}$$

This formula is almost always used in phenomenology

Why do we normally ignore DPS?

Crude formula for DPS cross section:

$$\sigma_D^{(A,B)} \approx \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

$\sigma_S^{(A)} \sim \frac{1}{Q^2}$, $\sigma_S^{(B)} \sim \frac{1}{Q^2}$, $\sigma_{eff} \sim \frac{1}{\Lambda^2}$

$$\Rightarrow \sigma_D^{(A,B)} = O\left(\frac{\Lambda^2}{Q^4}\right)$$

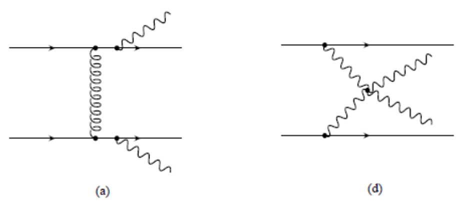
DPS is a power suppressed effect!

Why should we care about DPS at the LHC?

1. DPS can compete with SPS if SPS process is suppressed by small/multiple coupling constants:

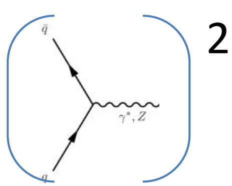
e.g. Same-sign WW production:

SPS:



$O(\alpha_s^2 \alpha_w^2)$ $O(\alpha_w^4)$

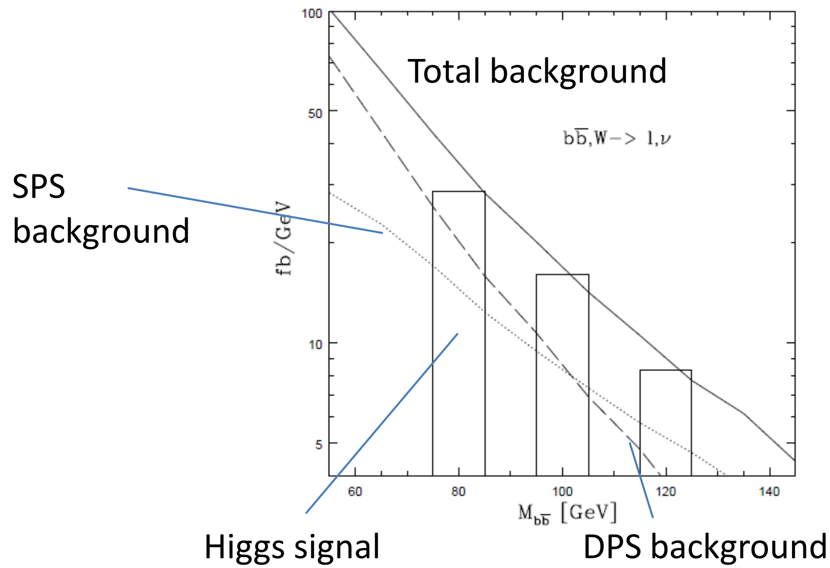
DPS:

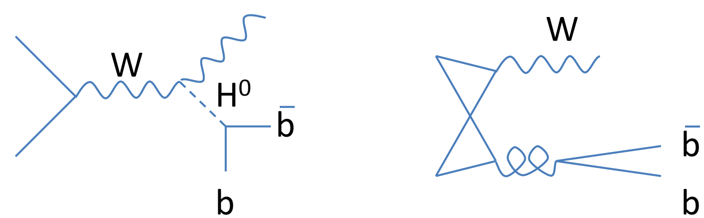


$O(\alpha_w^2)$

JG, Kom, Kulesza, and Stirling, Eur.Phys.J. C69 (2010) 53–65, Kulesza, and Stirling Phys. Lett. B475 (2000) 168–175.

e.g. H + W production:





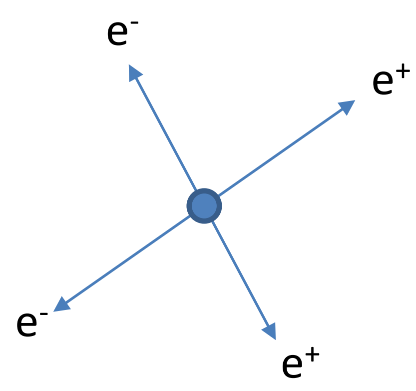
Del Fabbro and Treleani, Phys.Rev. D61 (2000) 077502. See also Bandurin et al., JHEP 1104 (2011) 054.

Why should we care about DPS at the LHC?

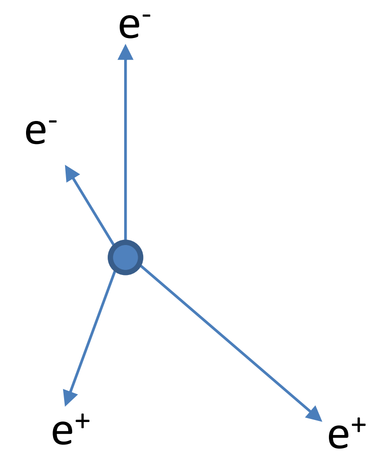
2. DPS populates the final state phase space in a different way from SPS:

e.g. $pp \rightarrow \underbrace{e^+ e^-}_{A} \underbrace{e^+ e^-}_{B}$

DPS:



SPS:



In the region with $\mathbf{q_A}$, $\mathbf{q_B}$ small, DPS and SPS are comparable for any process!

Why should we care about DPS at the LHC?

3. Consider again crude formula for DPS: $\sigma_D^{(A,B)} \approx \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$

For a given process, SPS cross section grows with collider energy because collider energy ↑ corresponds to x ↓, and PDF values are larger at small *x.*

DPS cross sections go like the product of SPS ones! ➔ DPS cross sections grow faster than SPS ones as the energy of the collider increases, and will be more important at the LHC than at any previous collider!

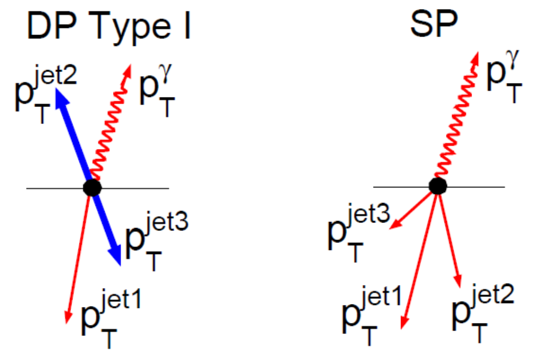
4. DPS reveals new information about the structure of the proton – in particular, correlations between partons in the proton (this information is not contained in PDFs, TMDs, GPDs, etc.).

Experimental Measurements of DPS

Experimental variables used to extract DPS from SPS rely on the previously-mentioned preference of DPS for 'pairwise back-to-back' events:

e.g. D0 γ+3j

$$S_{p'_T} = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{|\vec{p}_T(\gamma, i)|}{|\vec{p}_T^{\,\gamma}| + |\vec{p}_T^{\,i}|} \right)^2 + \left(\frac{|\vec{p}_T(j, k)|}{|\vec{p}_T^{\,j}| + |\vec{p}_T^{\,k}|} \right)^2},$$



DPS fraction often obtained by fitting SPS and DPS templates to experimental distributions, where SPS and DPS templates are generated using MC code (although D0 and CDF γ+3j experiments use a more data-driven method).

Overall distribution = $(1-f^{R}_{DP})$•Template A + f^{R}_{DP}•Template B

SPS **DPS fraction** **DPS**

Experimental Measurements of DPS

Experimental measurements more or less limited to σ_{eff} :

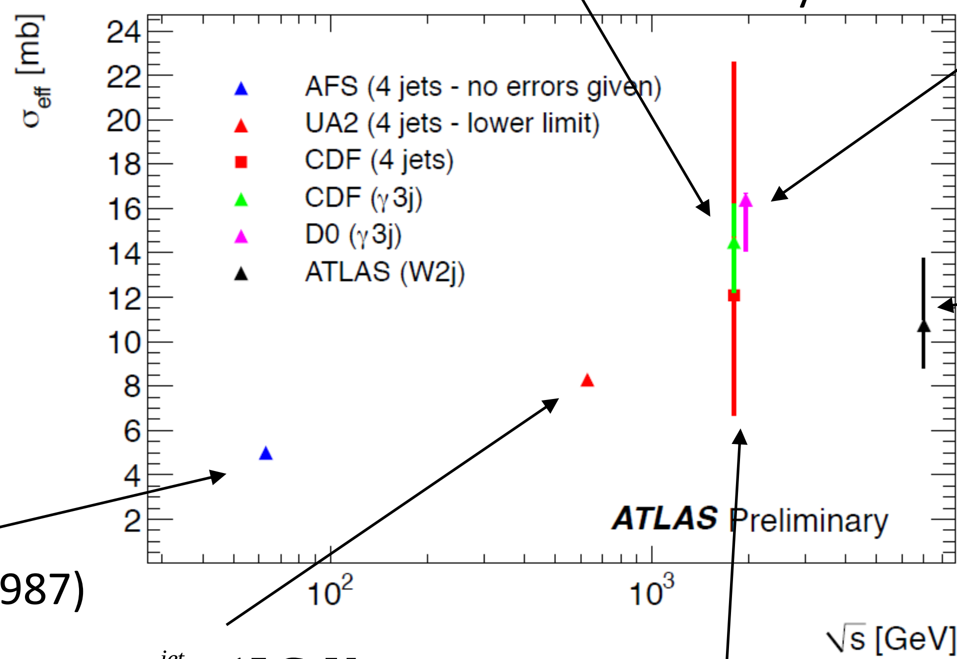
$p_T^{jet} > 6\,\text{GeV},\ p_T^{\gamma} > 16\,\text{GeV}$

Phys. Rev. D 56, 3811 (1997)

$15 < p_T^{jet2} < 30\,\text{GeV}$

$60 < p_T^{\gamma} < 80\,\text{GeV}$

Phys.Rev.D81, 052012(2010)



$p_T^{jet} > 20\,\text{GeV}$

$p_T^{lepton} > 20\,\text{GeV}$

ATLAS-CONF-2011-160

$p_T^{jet} > 4\,\text{GeV}$

Z. Phys. C 34, 163 (1987)

$p_T^{jet} > 15\,\text{GeV}$

Phys. Lett. B268, 145-154 (1991)

$p_T^{jet} > 25\,\text{GeV}$

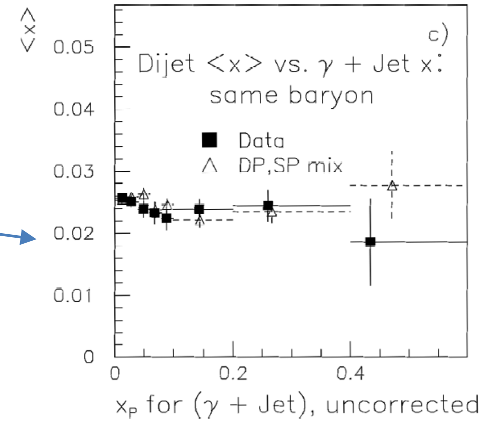
Phys. Rev. D 47, 4857-4871 (1993).

Experimental Measurements of DPS

Two exceptions to this trend in CDF and D0 studies of DPS contribution to γ+3 jets.

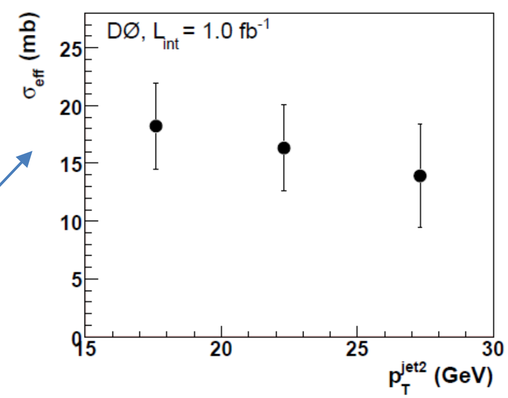
CDF investigated whether their data contained any evidence for *x* correlations between pairs of partons in the same proton. None found $\Rightarrow$ factorised approximation for dPDFs is good for sea quarks at low *x*.

$0.01 < x < 0.40$ for partons producing γj,
$0.002 < x < 0.20$ for partons producing jj.



Phys. Rev. D56 3811–3832, 1997.

D0 investigated variation of σ_{eff} with second largest jet p_T. Data consistent with no variation, although suggestion that ratio decreases with increase in p_T (effects of pQCD evolution on dPDFs?)



Phys. Rev. D81 052012, 2010.

Double PDF framework for calculating DPS

A quantity denoted as $D_h^{j_1 j_2}(x_1, x_2, Q^2)$ (the double PDF, or dPDF) was introduced in 1982 by Shelest, Snigirev and Zinovjev [Phys. Lett. B 113:325], and an evolution equation for this quantity was given (dDGLAP equation). Subsequently suggested [see e.g. Snigirev, Phys.Rev. D68 (2003) 114012] that this quantity is equal to the factorised longitudinal piece of the 2pGPD for the case where $Q_A = Q_B \equiv Q$.

Single DGLAP equation:

$$\frac{dD_h^j(x;t)}{dt} = \frac{\alpha_s(t)}{2\pi} \sum_{j'} \int_x^1 \frac{dx'}{x'} D_h^{j'}(x';t) P_{j' \to j}\left(\frac{x}{x'}\right)$$

Single PDF

Usual '1→1' splitting functions

Double DGLAP equation:

$$\frac{dD_h^{j_1 j_2}(x_1, x_2; t)}{dt} = \frac{\alpha_s(t)}{2\pi} \left[\sum_{j'_1} \int_{x_1}^{1-x_2} \frac{dx'_1}{x'_1} D_h^{j'_1 j_2}(x'_1, x_2; t) P_{j'_1 \to j_1}\left(\frac{x_1}{x'_1}\right) + \sum_{j'_2} \int_{x_2}^{1-x_1} \frac{dx'_2}{x'_2} D_h^{j_1 j'_2}(x_1, x'_2; t) P_{j'_2 \to j_2}\left(\frac{x_2}{x'_2}\right) + \sum_{j'} D_h^{j'}(x_1 + x_2; t) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}\left(\frac{x_1}{x_1 + x_2}\right) \right]$$

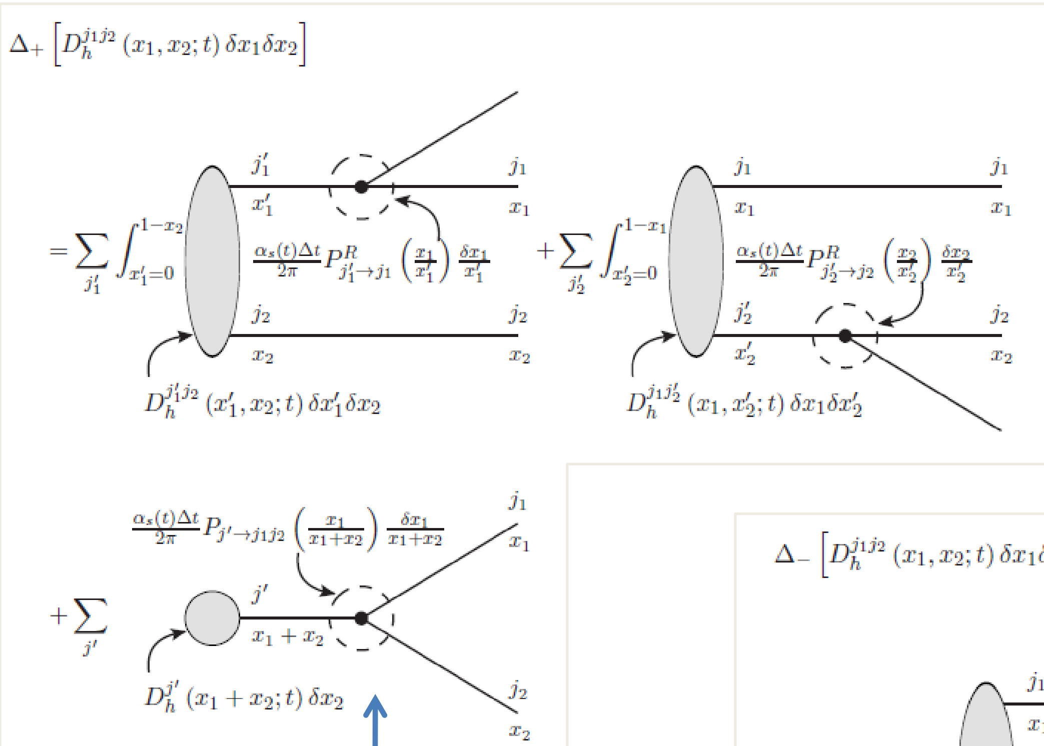
'Double PDF'

$$t \equiv \ln\left(Q^2\right)$$

Single PDF

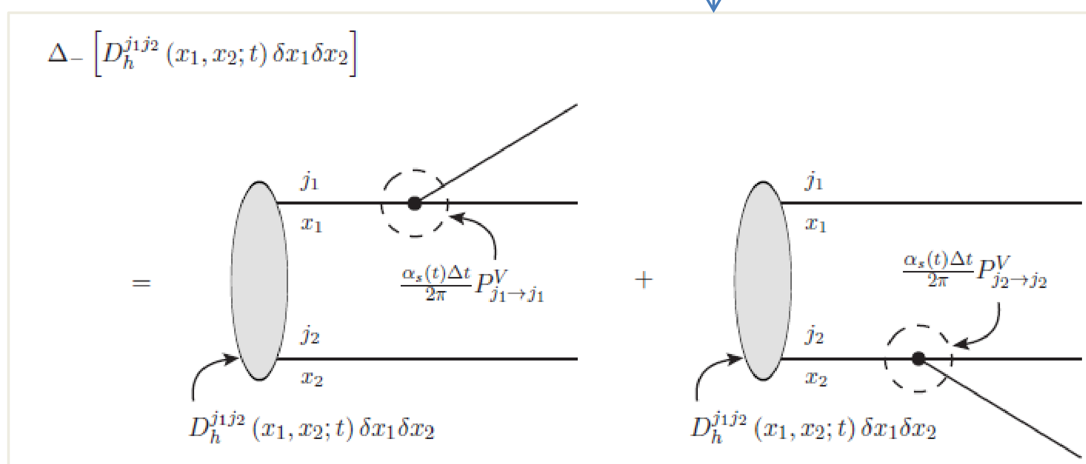
'1→2' splitting function NEW!

Pictorial representation of double DGLAP equation



Splitting processes acting to increase D^{ij} as the scale is increased from t $\rightarrow$ t $+\Delta$t.

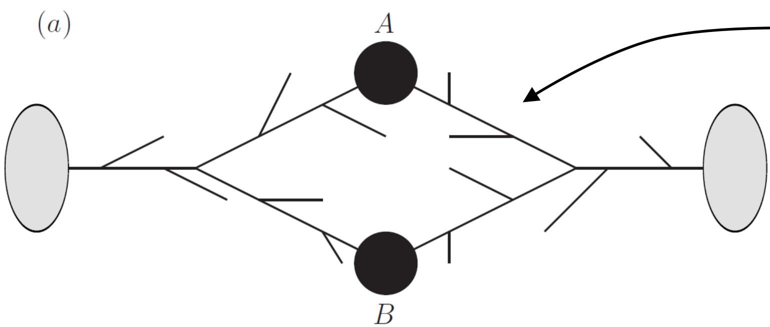
Splitting processes acting to decrease D^{ij} as the scale is increased from t $\rightarrow$ t $+\Delta$t.



“single PDF feed”

Double PDF framework for calculating DPS

$Q_A^2 = Q_B^2 = Q^2 > 0$



Given the inclusion of single feed term, dPDF framework predicts that part of these 'double perturbative splitting' or '1v1' graphs should be included as DPS. At the cross section level the part that should be included is proportional to:

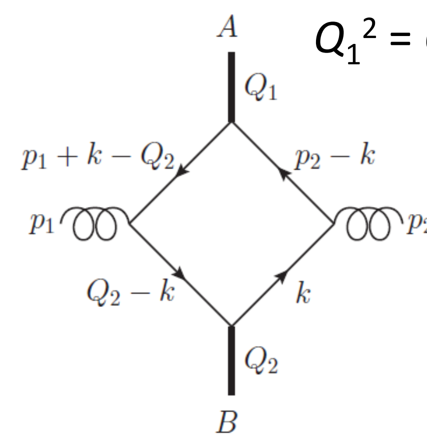
$$[\log(Q^2/\Lambda^2)]^n/\sigma_{eff}$$

n = total number of QCD branching vertices on either side of diagram.

This part should be associated with QCD branchings on either side of the diagram being strongly ordered in transverse momenta.

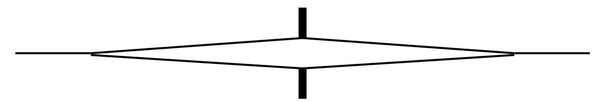
$Q_1^2 = Q_2^2 = Q^2 > 0$

Simplest example of a loop diagram with the given structure is the gg→AB crossed box diagram on the right. Does the cross section expression for this contain a piece proportional to $(\alpha_s \log(Q^2/\Lambda^2))^2/\sigma_{eff}$?



Double Parton Scattering Singularity

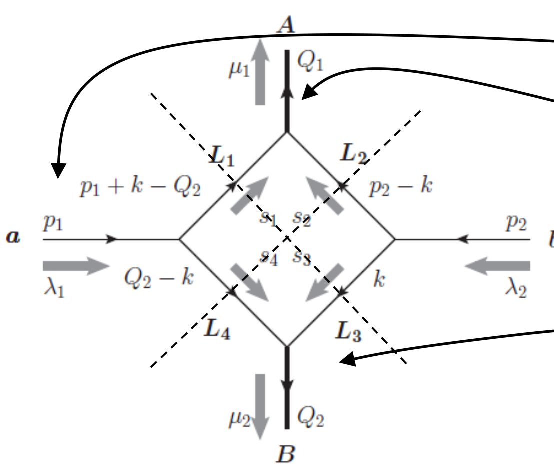
We would expect this piece to come from the portion of the external and loop integrations in which the transverse momenta of the outgoing particles are small, and all internal loop particles are almost on shell and collinear.



It is possible to obtain an analytic expression for the contribution to the loop from this region [JG and Stirling, JHEP 1106 048 (2011)]:

$$L_{DPS}(\lambda_1\lambda_2\mu_1\mu_2) = \sum_{s_i, L_i} \int d^d k \delta(k^2)\delta((k-Q_2)^2)\Phi^{\lambda_2 \to s_2 s_3}_{b \to L_2 L_3}(p_2; p_2 - k, k)$$
$$\times \Phi^{\lambda_1 \to s_1 s_4}_{a \to L_1 L_4}(p_1; p_1 + k - Q_2, Q_2 - k)\mathcal{M}^{s_3 s_4 \to \mu_2}_{L_3 L_4 \to B}(k, Q_2 - k; Q_2)$$
$$\times \mathcal{M}^{s_1 s_2 \to \mu_1}_{L_1 L_2 \to A}(p_1 + k - Q_2, p_2 - k; Q_1)$$

$$\left(\times\sqrt{Q_1^2 / Q_2^2}\right)$$

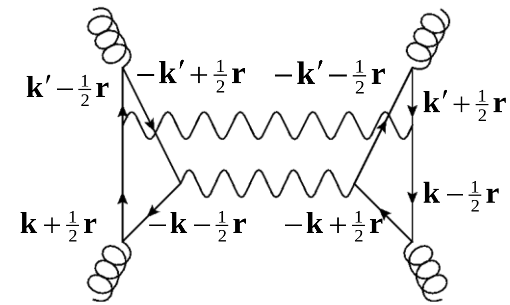


In SM, $\Phi^{\lambda_1 \to s_1 s_2}_{a \to bc}(p; k, k-p) = X^{\lambda_1 \to s_1 s_2}_{a \to bc}(x) K^{\lambda_1 \to s_1 s_2}_{a \to bc}(\mathbf{k})$

$K(\mathbf{k}) \sim 1/|\mathbf{k}|$ due to numerator factors

‘Double Perturbative Splitting’ graphs

Insert our analytic expression for DPS singular part of loop into standard 2 → 2 cross section formula:



$\boldsymbol{r}$ is Fourier conjugate of parton pair separation $\boldsymbol{b}$

$$\sigma_{DPS,\text{fig 1(b)}} \propto \int \prod_{i=1}^{2} dx_i d\bar{x}_i \hat{\sigma}_{q\bar{q}\to A}(\hat{s}=x_1\bar{x}_1 s)\hat{\sigma}_{q\bar{q}\to B}(\hat{s}=x_2\bar{x}_2 s)$$

$$\times \int \frac{d^2\boldsymbol{r}}{(2\pi)^2}\Gamma_{g\to q\bar{q}}(x_1,x_2,\boldsymbol{r})\Gamma_{g\to\bar{q}q}(\bar{x}_1,\bar{x}_2,-\boldsymbol{r})$$

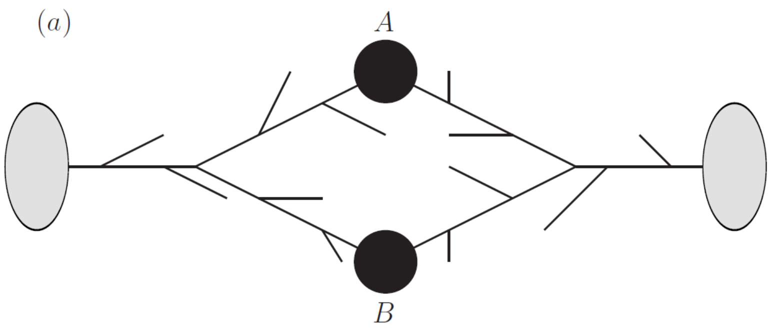
‘O(α_s) g → q$\bar{\text{q}}$ 2pGPD’

1 → 2 splitting function

$$\Gamma_{g\to q\bar{q}}(x_1,x_2,\boldsymbol{r}) \propto \frac{\alpha_S}{2\pi}\delta(1-x_1-x_2)T^{ij}(x_1,x_2)\int^{\tilde{\boldsymbol{k}}^2<\mathcal{O}(Q^2)} d^2\tilde{\boldsymbol{k}}\,\frac{[\tilde{\boldsymbol{k}}+\frac{1}{2}\boldsymbol{r}]^i[\tilde{\boldsymbol{k}}-\frac{1}{2}\boldsymbol{r}]^j}{[\tilde{\boldsymbol{k}}+\frac{1}{2}\boldsymbol{r}]^2[\tilde{\boldsymbol{k}}-\frac{1}{2}\boldsymbol{r}]^2}.$$

Obtain a result that is consistent with the double PDF framework if one considers the portion of the integral with $|\boldsymbol{r}| < \Lambda_S$ as DPS, where Λ_S is a specific choice of cut-off of the order of Λ_{QCD}. But why should we consider this piece specifically as DPS?

‘Double Perturbative Splitting’ graphs



Same issues are encountered for an arbitrary double perturbative splitting graph. There is no distinct piece of the arbitrary double splitting graph that contains a natural scale of order Λ_{QCD} and is associated with the transverse momenta inside the loop being strongly ordered on either side of the diagram. Most of the contribution to the cross section expression for this graph comes from the region of integration in which the transverse momenta of particles inside the loop are of $O(\sqrt{Q^2})$.

Perhaps, then, we shouldn’t include any of this graph as DPS. This has the advantage of avoiding potential double counting between DPS and SPS.

‘Double Perturbative Splitting’ graphs

There are clearly theoretical issues with the double PDF framework. The source of these problems can be exhibited by Fourier transforming the ***r***-space perturbative splitting 2pGPD into ***b*** space:

$$\Gamma_{qq}\Big|_{g\to q\bar{q}}\left(x_1, x_2, \mathbf{b}\right) \sim 1/\mathbf{b}^2$$

Power law behaviour – very different from smooth function of size R_p expected from double PDF framework. A key error in the formulation of the dPDF framework is the assumption that all 2pGPDs can be approximately factorised into dPDFs and smooth transverse functions of size R_p .

A sound theoretical framework for describing proton-proton DPS needs to carefully take account of the different ***b*** dependence of pairs of partons emerging from perturbative splittings, whilst simultaneously avoiding double counting between SPS and DPS.

See also Diehl and Schafer (Phys.Lett. B698 (2011) 389-402),
Diehl, Ostermeier and Schafer (JHEP 1203 (2012) 089).

Aside: Double PDFs

Does the concept of a 'double PDF' with only *x* arguments have any meaning?

Yes – we can just define the dPDF as the (regulated) integral of the 2pGPD over ***b*** (restrict ourselves to the case $Q_A = Q_B = Q$ here):

$$D_h^{p_1 p_2}(x_1, x_2; Q) \equiv \int d^2\mathbf{b} D_h^{p_1 p_2}(x_1, x_2, \mathbf{b}; Q, Q)$$

Is there any process in which the dPDFs are probed directly – i.e. in which they appear explicitly in the formula for the cross section?

The kind of process we are looking for is one in which two particles probe the proton's content, and those particles are uncorrelated in transverse space over length scales of the order of the proton radius.

Aside: Probing dPDFs

If the two probe particles come from different nucleons of a large *A* nucleus whose thickness does not vary much over the diameter of the proton, then we expect the ***b*** distribution of the pair of probe partons to be roughly uniform.

→ cross section expression for two-nucleon contribution to proton-nucleus DPS contains dPDF rather than 2pGPD:

Strikman and Treleani [Phys.Rev.Lett., 88:031801]

$$\sigma_2^D = \frac{1}{2}\int G_N(x_1,x_2)\hat{\sigma}(x_1,x_1')\hat{\sigma}(x_2,x_2')G_N(x_1')G_N(x_2')dx_1dx_1'dx_2dx_2'\int d^2BT^2(B)$$

$$G_N(x_1,x_2) = \int d^2b\Gamma_N(x_1,x_2;b)$$

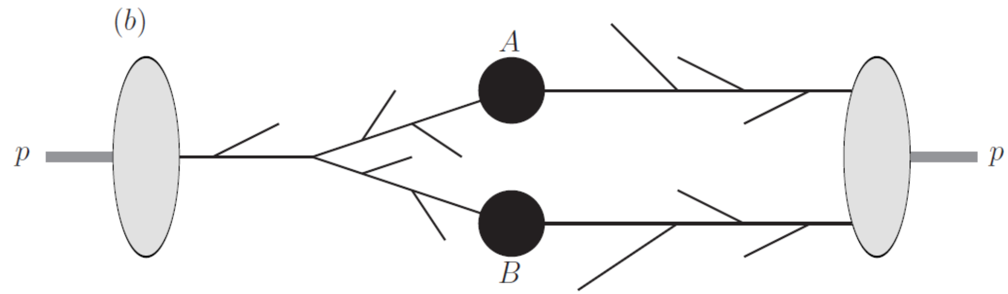
Nuclear thickness function

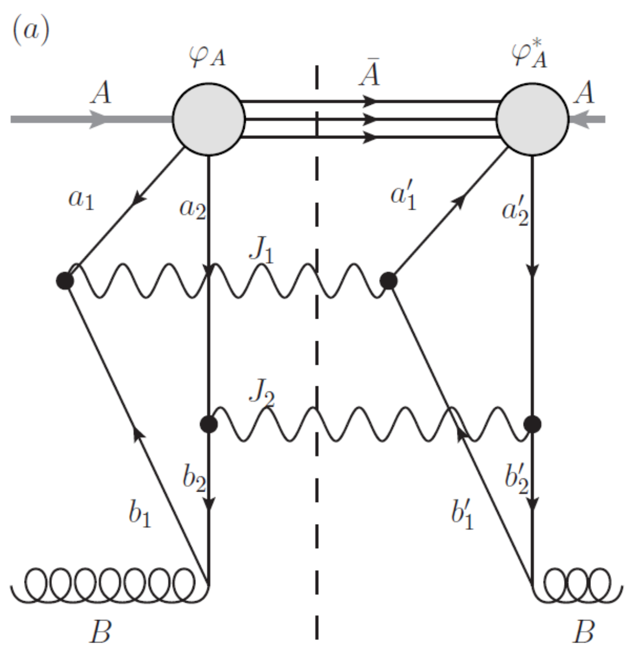
We have produced a set of dPDFs incorporating pQCD evolution effects via the dDGLAP equation, and momentum and number sum rule constraints – the 'GS09 dPDFs' (JG and Stirling, JHEP 1003 (2010) 005). These can be used in predictions of the two-nucleon contribution to proton-nucleus DPS.

Note that there is an object in the fragmentation region which evolves in a similar manner to the dPDFs – the 'double fragmentation function' $D_a^{h_1h_2}(x_1,x_2;t)$ - gives probability that a parton *a* emerging from a high *q* process will give rise to hadrons h_1 and h_2 + *X*.

What about the 2v1 contribution?

This is where one proton provides one parton to the double scattering, and the other two, at the nonperturbative level.





Take a similar approach as we did for the 1v1 graphs. Look at the simplest graph in which a single parton splits and then interacts with two 'nonperturbatively generated' partons from a proton, and see if there is a structure in the cross section formula ~ $\log(Q^2/\Lambda^2)/R_p^2$

Need to use a wavefunction on the side with the two nonperturbative partons to represent the fact that the two partons are tied together in the same proton.

What about the 2v1 contribution?

Result:

Required large logarithm

$$\sigma_{1v2}(s) = \sum_{s_i s_i' t_i t_i'} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{\bar{q}q\to\gamma*}^{s_1,t_1;s_1',t_1';\mu_1}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{q\bar{q}\to\gamma*}^{s_2,t_2;s_2',t_2';\mu_2}(\hat{s} = x_2 y_2 s)$$

$$\times \quad \Gamma_A^{s_1 s_2, s_1' s_2'}(x_1, x_2; \boldsymbol{b} = \boldsymbol{0}) \quad \left[\frac{\alpha_s}{2\pi} P_{g\to q\bar{q}}^{\lambda\to t_2 t_1, t_2' t_1'}(y_2)\,\delta(1 - y_1 - y_2) \int_{\Lambda^2}^{Q^2} \frac{d\boldsymbol{J_1}^2}{\boldsymbol{J_1}^2}\right]$$

2pGPD of nonperturbatively generated parton pair evaluated at **b** = **0**

1 ⟶ 2 splitting function

Summing leading logarithmic parts of all 2v1 graphs (diagonal unpolarised contribution):

$$\sigma_{(A,B)}^{D,1v2}(s) = 2 \times \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2 y_2 s)$$

$$\times\, \breve{D}_p^{ij}(x_1, x_2; Q^2)\Gamma_{p,indep}^{kl}(y_1, y_2, \boldsymbol{b} = \boldsymbol{0}; Q^2)$$

'sPDF feed' part of dPDF

'Independent branching' 2pGPD

Agrees with 2v1 contribution to DPS cross section recently proposed by Ryskin and Snigirev (Phys.Rev. D83 (2011) 114047), and 2v1 contribution in equation (11) of Blok et al., [arXiv:1106.5533].

What about the 2v1 contribution?

The critical requirement for the validity of the derivation on the previous page is that parton pairs connected only via nonperturbative interactions should have an ***r*** distribution that is cut off at values of order Λ_{QCD} (or a ***b*** distribution that is smooth on scales of size $<< R_p$). That is, the ***r*** profile of $\Gamma^{kl}_{p,indep}(y_1, y_2, \mathbf{\Delta}; Q^2)$ should have a width of order Λ_{QCD} .

The results of the previous slide are potentially misleading, in that they appear to indicate that 2v1 contribution to DPS probes independent branching 2pGPDs at zero parton separation. In fact, the results correspond to values of $\boldsymbol{b}^2$ that are $<< R_p^2$ but $>> 1/Q^2$.

If we assume $\Gamma^{ij}_{p,indep}(x_1, x_2, \boldsymbol{b}; Q^2) = \tilde{D}^{ij}(x_1, x_2; Q^2) F(\boldsymbol{b})$ then 2v1 contribution to DPS cross section is similar to that predicted by dPDF framework, except with a different 'σ_{eff}' :

$$\frac{1}{\sigma_{eff,2v2}} \equiv \int d^2\boldsymbol{b}[F(\boldsymbol{b})]^2$$

$$\frac{1}{\sigma_{eff,1v2}} \equiv \quad F(\boldsymbol{b} = \boldsymbol{0})$$

Naive Gaussian for F(**b**) gives factor of two enhancement for 2v1. F(**b**) is nonperturbative however – don't really know a lot about it.

The DPS Cross Section

Combining suggestions for 1v1 and 2v1 graphs, we obtain the following formula for the DPS cross section:

$$\sigma^{D}_{(A,B)}(s) = \sigma^{D,2v2}_{(A,B)}(s) + \sigma^{D,1v2}_{(A,B)}(s)$$

$$\sigma^{D,1v2}_{(A,B)}(s) = 2 \times \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2 y_2 s)$$
$$\times \breve{D}^{ij}_{p}(x_1, x_2; Q^2)\Gamma^{kl}_{p,indep}(y_1, y_2, \boldsymbol{b} = \boldsymbol{0}; Q^2)$$

$$\sigma^{D,2v2}_{(A,B)}(s) = \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2 y_2 s)$$
$$\times \int d^2\boldsymbol{b}\,\Gamma^{ij}_{p,indep}(x_1, x_2, \boldsymbol{b}; Q^2)\Gamma^{kl}_{p,indep}(y_1, y_2, \boldsymbol{b}; Q^2)$$

The DPS Cross Section

Comments on this formula:

1. We were originally expecting to get a formula for the DPS cross section looking something like:

$$\sigma^D_{(A,B)} \propto \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, \boldsymbol{b}; Q_A^2, Q_B^2)\hat{\sigma}_{ik\to A}(\hat{s} = x_1 x_3 s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2 x_4 s)$$
$$\times \Gamma_{kl}(x_3, x_4, \boldsymbol{b}; Q_A^2, Q_B^2) dx_1 dx_2 dx_3 dx_4 d^2\boldsymbol{b}$$

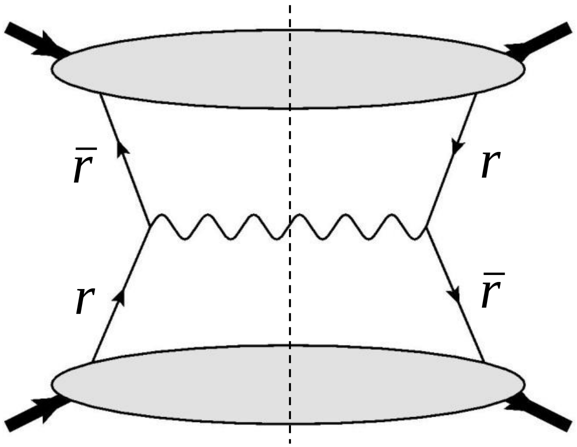
with the 2pGPDs being expressible in terms of hadronic matrix elements. What we have got does not seem to look like this.

2. In this formula, we have made a sharp distinction between perturbatively and nonperturbatively generated parton pairs. Is there some scale at which we can regard all parton pairs in the proton as being 'nonperturbatively generated', and if so what is the appropriate choice for this scale? (Presumably something rather close to Λ_{QCD}).

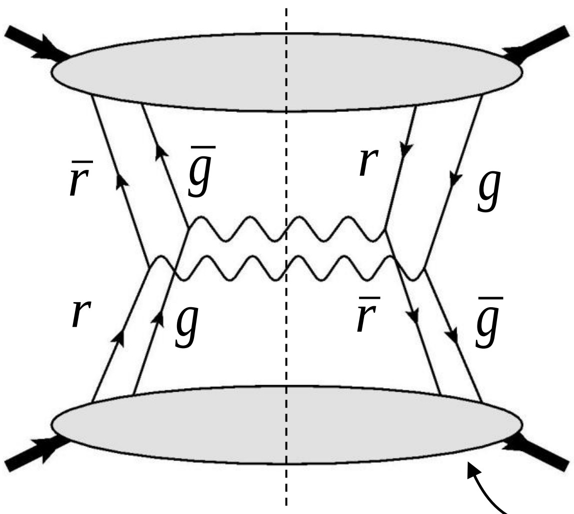
and there's something else we've so far not discussed...

Interference contributions to proton-proton DPS

In proton-proton SPS, only one parton 'leaves' each proton, interacts, and then returns.



➔ parton must return with the same quantum numbers it left with to reform proton ➔ no interference.

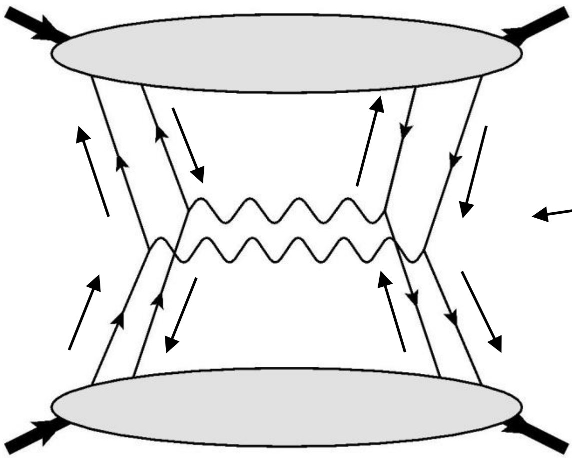


In proton-proton DPS, fact that interacting partons must recombine with spectators to form original proton only imposes conditions on overall quantum numbers of diparton system

➔ possibility of interference diagrams in which discrete quantum numbers are swapped between the two partons in going from the LHS to RHS.

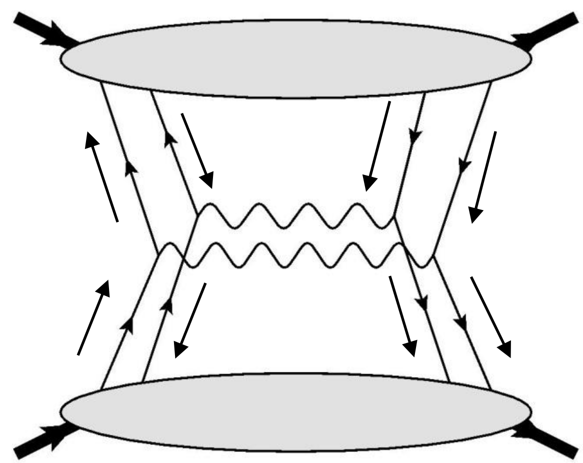
Example for colour.

Interference contributions to proton-proton DPS (spin)



Can clearly have the analogous diagrams to the ones that are allowed for colour.

This probes 'double transversity' distributions δqδq



For $\mathbf{b} \neq \mathbf{0}$ diparton system can have orbital angular momentum, which can be different between the LHS and RHS → diagrams that don't conserve helicity can also contribute.

This probes 'single transversity' distributions qδq

Polarised PDF contributions to proton-proton DPS

In proton-proton DPS, there exists the possibility of having contributions to the cross section associated with polarized 2pGPDs, even when the colliding protons are unpolarized!

Reason for this: there may be correlations in helicity between the two active partons!

e.g.

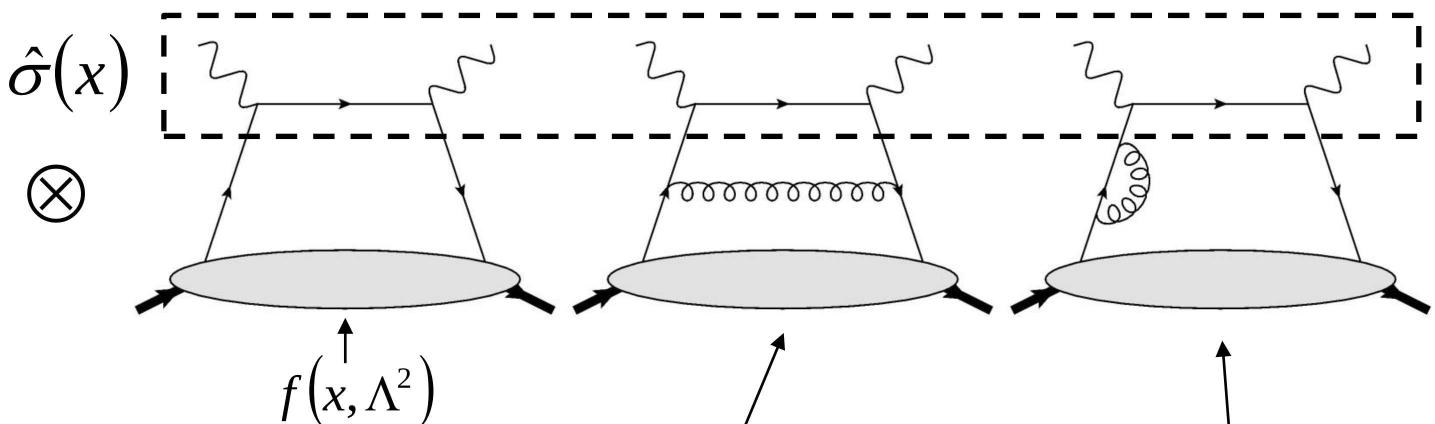
$$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

If probability to find two quarks with same spin differs from probability to find two quarks with opposing spins, $\Delta q_1 \Delta q_2 \neq 0$.

Similarly – contributions associated with colour correlations between partons.

Issues of interference & spin/colour correlations discussed in more technical detail in Mekhfi (Phys.Rev. D32 (1985) 2380), Diehl, Ostermeier and Schafer (JHEP 1203 (2012) 089), Manohar and Waalewijn (arXiv:1202.3794).

Sudakov Suppression of Colour Interference/Correlation Distributions



Ellis, Stirling, Webber book, Chapter 5

$$+\frac{\alpha_s}{2\pi}C_R\ln\left(\frac{Q^2}{\Lambda^2}\right)\int_0^{1-\Lambda^2/Q^2}\frac{dx'}{x'}\frac{1+x'^2}{1-x'}\,f\left(\frac{x}{x'},\Lambda^2\right)$$

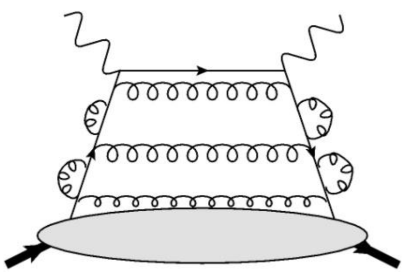
$$-\frac{\alpha_s}{2\pi}C_V\ln\left(\frac{Q^2}{\Lambda^2}\right)f\left(x,\Lambda^2\right)\int_0^{1-\Lambda^2/Q^2}dx'\frac{1+x'^2}{1-x'}$$

$$+\frac{\alpha_s}{\pi}C_R\ln^2\left(\frac{Q^2}{\Lambda^2}\right)f\left(x,\Lambda^2\right)$$

$$-\frac{\alpha_s}{\pi}C_V\ln^2\left(\frac{Q^2}{\Lambda^2}\right)f\left(x,\Lambda^2\right)$$

Sudakov Suppression of Colour Interference/Correlation Distributions

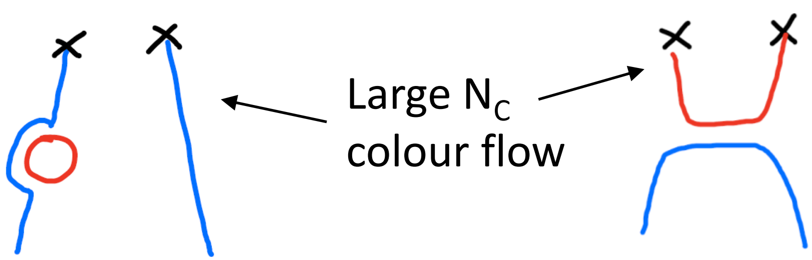
Sum up arbitrary number of real & virtual emissions to double log order:



Sudakov factor

$$\sigma = \hat{\sigma}(x) \otimes f\left(x, \Lambda^2\right) \exp\left(\frac{\alpha_s}{\pi} \left(C_R - C_V\right) \ln^2\left(\frac{Q^2}{\Lambda^2} \right) \right)$$

Quark legs are in colour singlet:



$C_V = C_F$

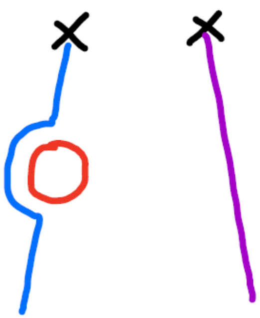
$C_R = C_F$

$C_R = C_V$

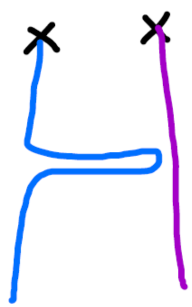
There is no Sudakov suppression!

Sudakov Suppression of Colour Interference/Correlation Distributions

Quark legs are in colour octet (as occurs in colour interference/correlation distributions):



$$C_V = C_F$$



$$C_R = \left(C_F - \frac{1}{2} C_A \right) = -\frac{1}{2N}$$

$$C_R - C_V < 0$$

➔ Strong Sudakov suppression of colour interference/correlation distributions!

Physically, Sudakov suppression is associated with the fact that colour interference distributions involve movement of colour by the large transverse distance **b** in the hadron.

Appropriate low scale cut-off in Sudakov factor for colour interference/correlation distributions is probably $1/\mathbf{b}^2$ rather than Λ^2 – soft gluons with wavelengths > |**b**| cannot resolve the transverse colour transfer.

First shown in Mekhfi and Artru, Phys.Rev. D37 (1988) 2618, and revisited in Diehl, Ostermeier and Schafer (JHEP 1203 (2012) 089) and Manohar and Waalewijn (arXiv:1202.3794).

Summary

- DPS can be important as a background to rare (Higgs/new physics) signals at the LHC. Also interesting as a signal – gives info on correlations in parton pairs. In $|\mathbf{q_A}|, |\mathbf{q_B}| \sim \Lambda$ region DPS and SPS are comparable.
- We showed that '1v1 graphs' appear to be included in the DPS cross section in an incorrect way in the 'dPDF framework' for calculating DPS. Maybe we should consider all of such graphs as SPS?
- Calculation of a simple 2v1 graph seems to indicate that 2v1 diagrams should be included in DPS cross section, but with a different geometrical prefactor.
- Is the total cross section then just a sum of 2v2 and 2v1 contributions, with different σ_{eff} for each?
- There are interference and polarised contributions to DPS, even in unpolarised pp scattering. However, colour interference/correlation distributions are Sudakov suppressed.

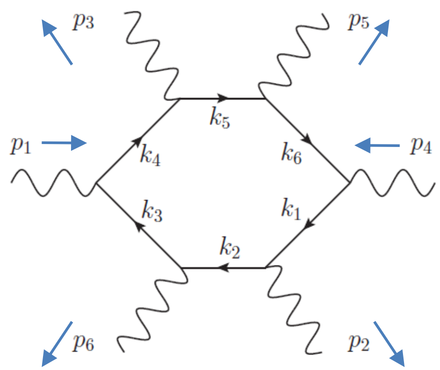
Backup Slides

Aside: DPS divergences in Six Photon Amplitude

Our analytical expression for the DPS divergence of a one-loop diagram can be used to **explain** interesting behaviour of amplitudes around DPS singular points that has been observed using ‘traditional’ NLO multileg integration techniques.

e.g. Six photon amplitude

Take all helicities as incoming, label helicity amplitude as $\lambda_1\lambda_2\ \lambda_3\lambda_4\ \lambda_5\lambda_6$



This is just one diagram contributing to the amplitude, which has a DPS singularity at $\mathbf{P}_\Sigma = \mathbf{p}_3 + \mathbf{p}_5 = 0$

Detailed numerical studies of specific MHV and NMHV amplitude by Bernicot and Guillet revealed the following properties of these amplitudes:

1. Neither helicity amplitude diverges as $\mathbf{P}_\Sigma \rightarrow 0$ like $1/\mathbf{P}_\Sigma^2$, as was expected by some authors.
2. The NMHV $- - - +++$ amplitude is finite at $\mathbf{P}_\Sigma = 0$.
3. The MHV $- ++ - ++$ amplitude is also finite at $\mathbf{P}_\Sigma = 0$.

Bernicot, arXiv:0804.1315,
Bern et. al., arXiv:0803.0494

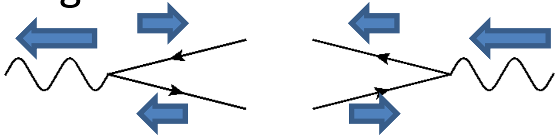
Aside: DPS divergences in Six Photon Amplitude

1. No helicity amplitude diverges at $\mathbf{P}_\Sigma \rightarrow 0$ like $1/\mathbf{P}_\Sigma^2$, as was expected by some authors.

Already explained. Associated with angular momentum nonconservation at both $\gamma \longrightarrow q\bar{q}$ vertices in collinear limit.

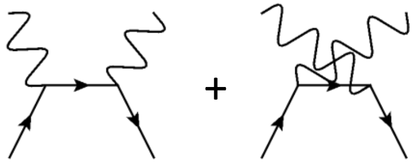
2. The NMHV $- - -$ +++ amplitude is finite at $\mathbf{P}_\Sigma = 0$.

Overall J_z nonconservation between $\gamma\gamma$ initial state and $q\bar{q}q\bar{q}$ intermediate state in collinear limit weakens DPS divergence such that it is finite.



3. The MHV $-$ ++ $-$ ++ amplitude is perfectly finite at $\mathbf{P}_\Sigma = 0$.

There are four graphs giving a DPS divergence at the point $\mathbf{P}_\Sigma = 0$. The matrix elements to be used in the calculation of the DPS divergent parts of the **sum** of these graphs are the sum of the following two graphs:



= full matrix element for $q\bar{q} \longrightarrow \gamma\gamma$. For MHV amplitude studied, photons have same helicity in both matrix elements, and go to zero by MHV rules for QED.

Brief Interlude – PDFs and TMDs

To make theoretical predictions in the small $\mathbf{q_A}$, $\mathbf{q_B}$ region, require the 'TMD 2pGPDs' F($x_{1,}$ $x_{2,}$ **b**, $\mathbf{k_1}$, $\mathbf{k_2}$, Q^2) – will not talk about these in detail in this talk, rather I will focus on total DPS cross section and 2pGPDs.

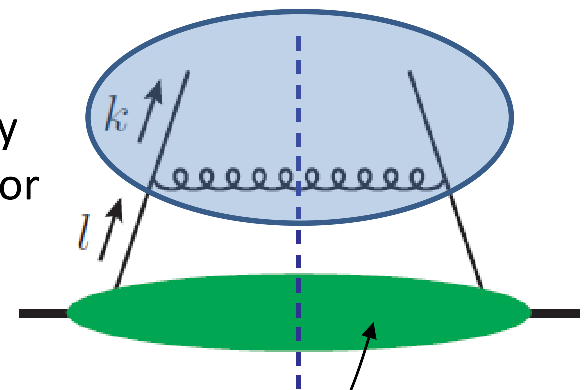
Note however that there is a connection between collinear PDFs and TMDs in single scattering case for $\Lambda^2 << \mathbf{q^2} << Q^2$ – there should be a similar connection between TMD 2pGPDs and 2pGPDs.

$$F_h(x, \mathbf{k}) =$$

$$T(x, \mathbf{k})$$

Perturbatively calculable factor

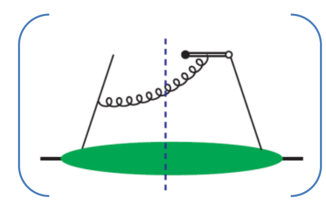
$$\otimes$$



$$D_h(x, \mu^2 = \mathbf{k}^2)$$

Collinear (single) PDF

+



*Note – I've omitted dependencies on the rapidity regulator ζ.

DPS singularities in covariant gauges

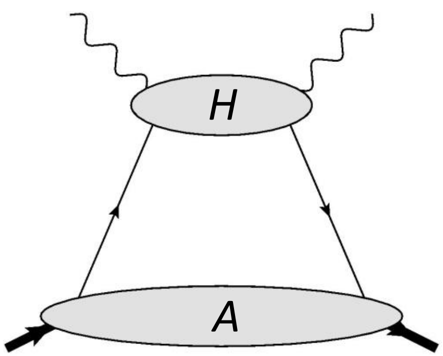
To reach these conclusions we've used a physical gauge – how do things change in a covariant gauge (such as Feynman gauge)?

In a covariant gauge, gluons with unphysical `scalar' polarisation can exist in loop diagrams. These can give rise to power-law DPS divergences rather than logarithmic ones in individual diagrams, and additional `super-leading' contributions to the AB production process (in terms of powers of Q).

On the other hand one generally expects the `super-leading' contribution to cancel in a suitable sum over graphs, as happens in SPS.

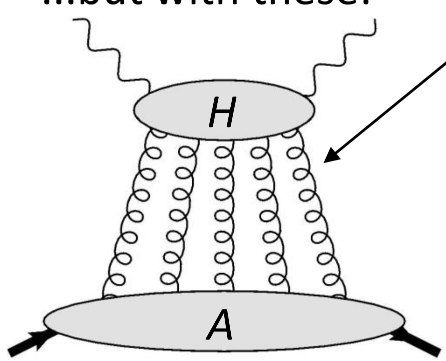
e.g. DIS

Highest power of Q is not actually associated with these graphs...



$O(Q^0)$

...but with these:



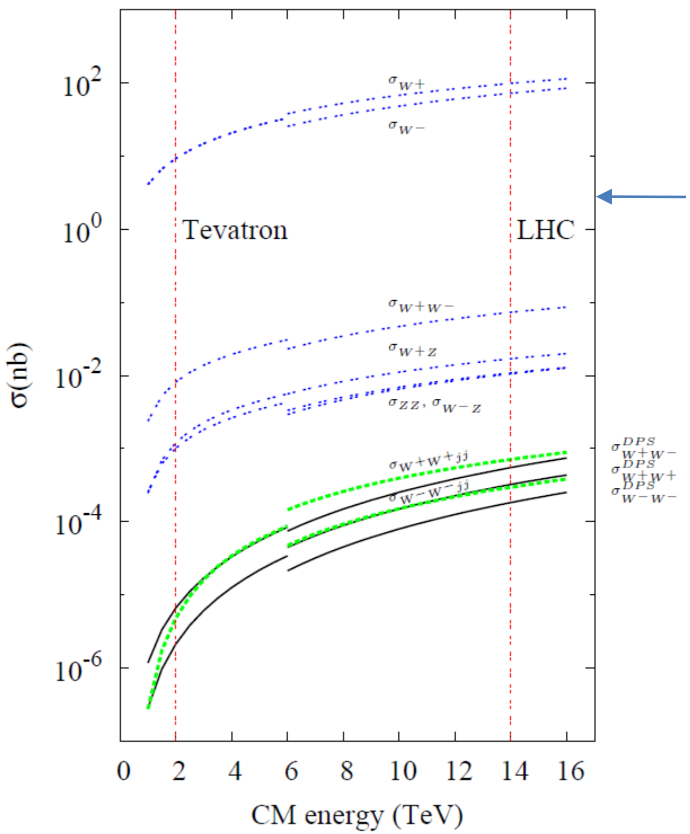
$O(Q^2)$

All gluons scalar polarised

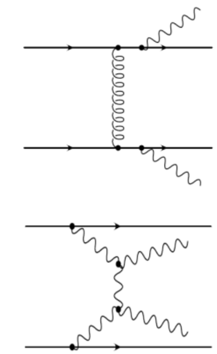
But super-leading behaviour of these graphs is cancelled when one sums over gluon attachments to H!

Labastida and Sterman, Nucl. Phys. B 254 (1985) 425.

Possibility of observing SSWW DPS at LHC

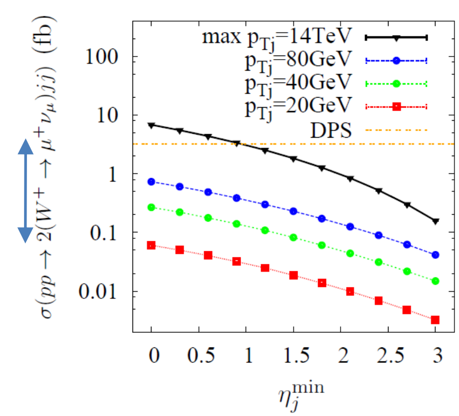


SPS same-sign WW production is forbidden at order $\alpha_W^2 \Rightarrow \sigma$ for this process is comparable to DPS σ, and always involves 2j.



This SPS background can be efficiently removed via a jet veto.

~2 orders of magnitude!



Other backgrounds to SSWW DPS

There are other SPS processes that can mimic the DPS same sign lepton signal:

•Heavy flavour

$$gg \to b\bar{b} \to B\bar{B} + \dots\,, \qquad t \to W^+ b \to l^+ \nu b,$$

$$B \to l^+ \nu X, \qquad \bar{t} \to W^- \bar{b} \to q\bar{q}' l^+ \nu c.$$

$$\bar{B}^0 \to B^0 \to l^+ \nu \tilde{X},$$

•Electroweak gauge boson pair

$$Z(\gamma^*)Z(\gamma^*) \to l^+(l^-)l^+(l^-) \qquad W^+Z(\gamma^*) \to l^+\nu l^+(l^-)$$

(If these are not detected)

Thus this channel is not as 'clean' with regards to DPS as had been previously thought – carefully chosen cuts required to enhance S/B sufficiently.

	$\sigma_{\mu^+\mu^+}$ (fb)	$\sigma_{\mu^-\mu^-}$ (fb)
$W^\pm W^\pm$(DPS)	0.82	0.46
$W^\pm Z(\gamma^*)$	5.1	3.6
$Z(\gamma^*)Z(\gamma^*)$	0.84	0.67
$b\bar{b}$ ($p_T^b \geq 20$ GeV)	0.43	0.43

cuts

$$\sigma_{b\bar{b}}(p_T \geq 20 \text{ GeV}) = 5.15\ \mu\text{b}$$