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The GS09 Double Parton Distribution Functions

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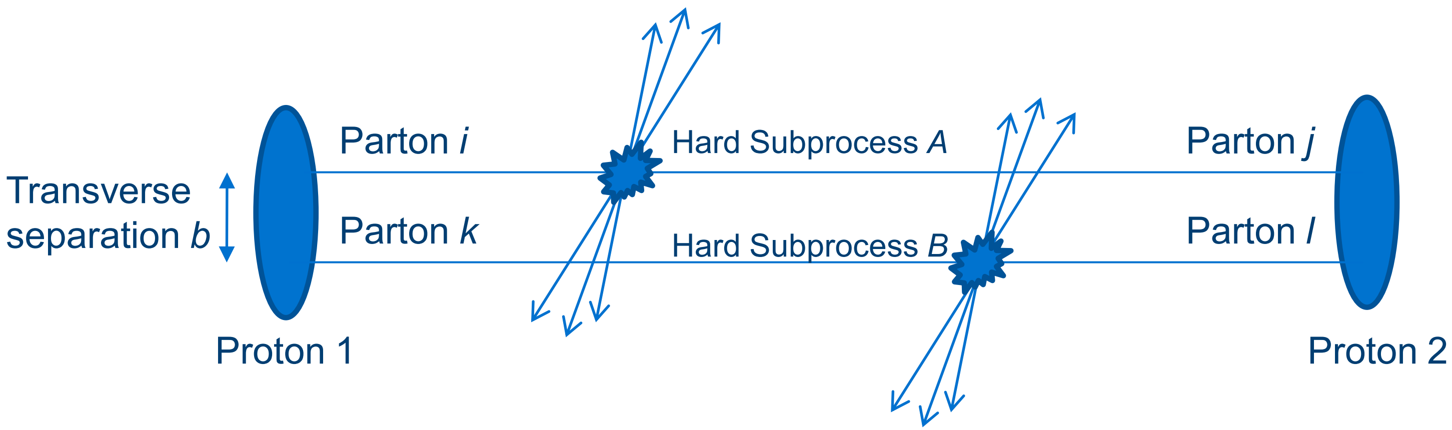
Work performed in collaboration with W.J. Stirling (arXiv:0910.4347), and C.H. Kom, A. Kulesza and W.J. Stirling (arXiv:1003.3953).

Outline

- Introduction: double parton scattering, double PDFs and double DGLAP.
- The number and momentum sum rules for the dPDFs.
- Development of the GS09 dPDFs, focussing on the design of input distributions approximately satisfying the sum rules.
- Difference between GS09 dPDFs and factorised forms previously used, in the context of same-sign WW DPS signal.
- Summary.

Double Parton Scattering (DPS)



Assuming factorisation of the two hard subprocesses:

$$\sigma_D^{(A,B)} = \underbrace{\frac{m}{2}}_{\text{Symmetry factor}} \sum_{i,j,k,l} \int \overbrace{\Gamma_h^{ik}\left(x_1, x_2, b; Q_A, Q_B\right) \Gamma_h^{jl}\left(x_1', x_2', b; Q_A, Q_B\right)}^{\text{Generalised double distributions}}$$

$$\times \underbrace{\hat{\sigma}_{ij}^{A}\left(x_1, x_1'\right) \hat{\sigma}_{kl}^{B}\left(x_2, x_2'\right)}_{\text{Parton level cross sections}} \mathrm{d}x_1 \, \mathrm{d}x_1' \mathrm{d}x_2 \, \mathrm{d}x_2' \mathrm{d}^2 b$$

Simplifying assumptions for DPS Cross Section

1. Take Γ to be a product of longitudinal and transverse pieces .

$$\Gamma_h^{ik}(x_1, x_2, b; Q_A, Q_B) = D_h^{ik}(x_1, x_2; Q_A, Q_B)\, F_k^i(b)$$

Double parton distribution functions (dPDFs)

Parton pair density in transverse space

2. Assume that F does not depend on parton indices – i.e. $F_k^i(b) = F(b)$

Then, if we define $\sigma_{eff} = \dfrac{1}{\int [F(b)]^2 d^2 b}$ we may write DPS cross section as:

$$\sigma_D^{(A,B)} = \frac{m}{2\sigma_{eff}} \sum_{i,j,k,l} \int D_h^{ik}(x_1, x_2; Q_A, Q_B) D_h^{jl}(x_1', x_2'; Q_A, Q_B)$$
$$\times \hat{\sigma}_{ij}^A(x_1, x_1') \hat{\sigma}_{kl}^B(x_2, x_2') \mathrm{d}x_1 \mathrm{d}x_1' \mathrm{d}x_2 \mathrm{d}x_2'$$

3. Neglect longitudinal correlations $D_h^{ij}(x_1, x_2; Q_A, Q_B) \approx D_h^i(x_1; Q_A) D_h^j(x_2; Q_B)$

Generally thought to be a good approximation at low x_i – though we have shown that this is not the case for all dPDFs (see later).

$$\Rightarrow \sigma_D^{(A,B)} \approx \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

Why should we care about DPS at the LHC?

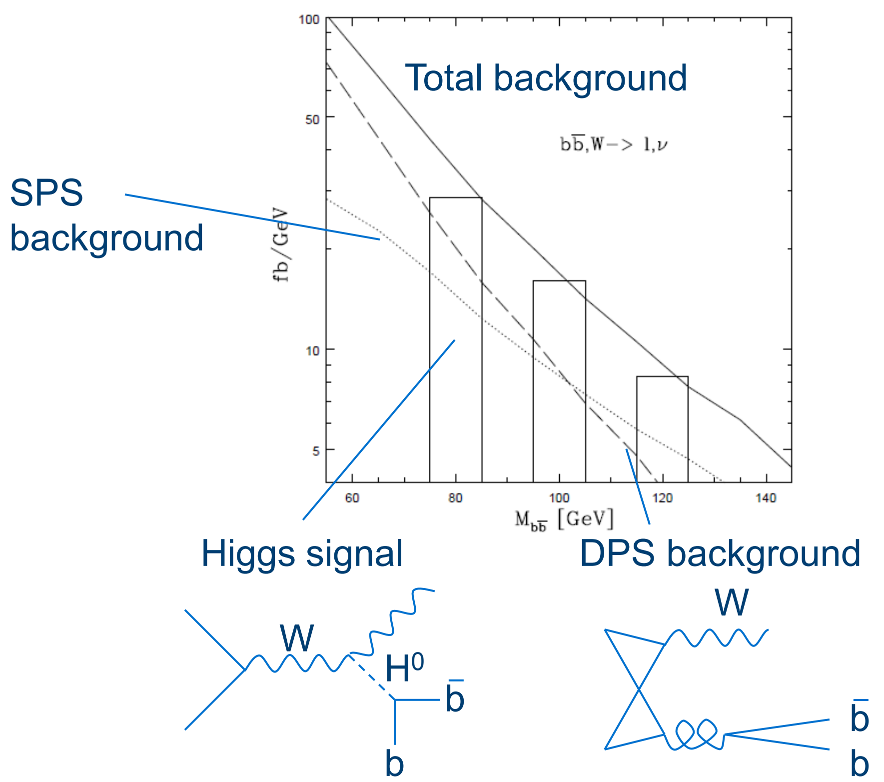
Consider crudest approximation for DPS cross section:

$$\sigma_D^{(A,B)} \approx \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

⇒DPS cross sections go like the product of SPS ones!
⇒DPS cross sections **grow faster with energy than SPS** $\boldsymbol{\sigma}$**.**

DPS processes...
- provide significant backgrounds to Higgs and new physics signals.
- reveal information about the structure of the proton.

DPS background to Higgs + W production (Del Fabbro and Treleani, hep-ph/9911358,1999):



Theoretical Ingredients of DPS Cross Section

$$\hat{\sigma}_{ij}^{A}\left(x_1, x_1'\right)$$

Parton-level cross sections

•Known for essentially all processes of phenomenological interest

$$\sigma_{eff}$$

Effective Cross Section

•Non-perturbative.

•May actually depend on parton indices, hard scales and/or x_i due to partial violation of earlier assumptions 1 and 2 – unlikely to vary much with these variables however.

•Measured as $\sigma_S^{(A)}\sigma_S^{(B)}/\sigma_D^{(A,B)}$ by CDF and D0 collaborations, in an interaction and x_i region for which we are confident that the above ratio reliably gives σ_{eff}.

•We use the CDF value in our phenomenological studies (14.5mb) with the caveat that it should be re-measured at the LHC using appropriate benchmark processes.

$$D_h^{ik}\left(x_1, x_2; Q_A, Q_B\right)$$

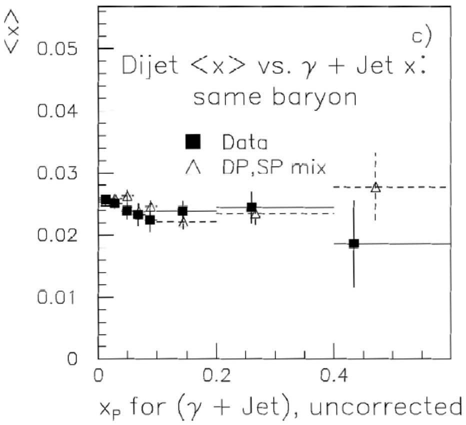
Double parton distribution functions – **what do we know about these**?

Experimental Studies

CDF and D0 studies of DPS contribution to γ+3 jets (A = 2*j*, B = γj).

Majority of their events were produced by low *x* sea parton collisions.
For CDF sample, $0.01 < x < 0.40$ for partons producing γj, and $0.002 < x < 0.20$ for partons producing *jj*.

CDF investigated whether their data contained any evidence for *x* correlations between pairs of partons in the same proton. None found $\Rightarrow$ factorised approximation for dPDFs is good for sea partons at low *x*.



Phys. Rev. D56 3811–3832, 1997.

Theoretical Work

Kirschner, Shelest, Snigirev, and Zinovjev have derived a ‘double DGLAP equation’ describing the change in the dPDFs with factorisation scale, for the special case of the dPDFs with $Q_A = Q_B = Q$.

$$\frac{dD_h^{j_1 j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\Bigg[\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1' j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right)$$

$$+\sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1 j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right)$$

$$+\sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1 j_2}\left(\frac{x_1}{x_1+x_2}\right)\Bigg]$$

$$\boxed{t = \ln\left(Q^2\right)}$$

Usual 1→1 splitting functions

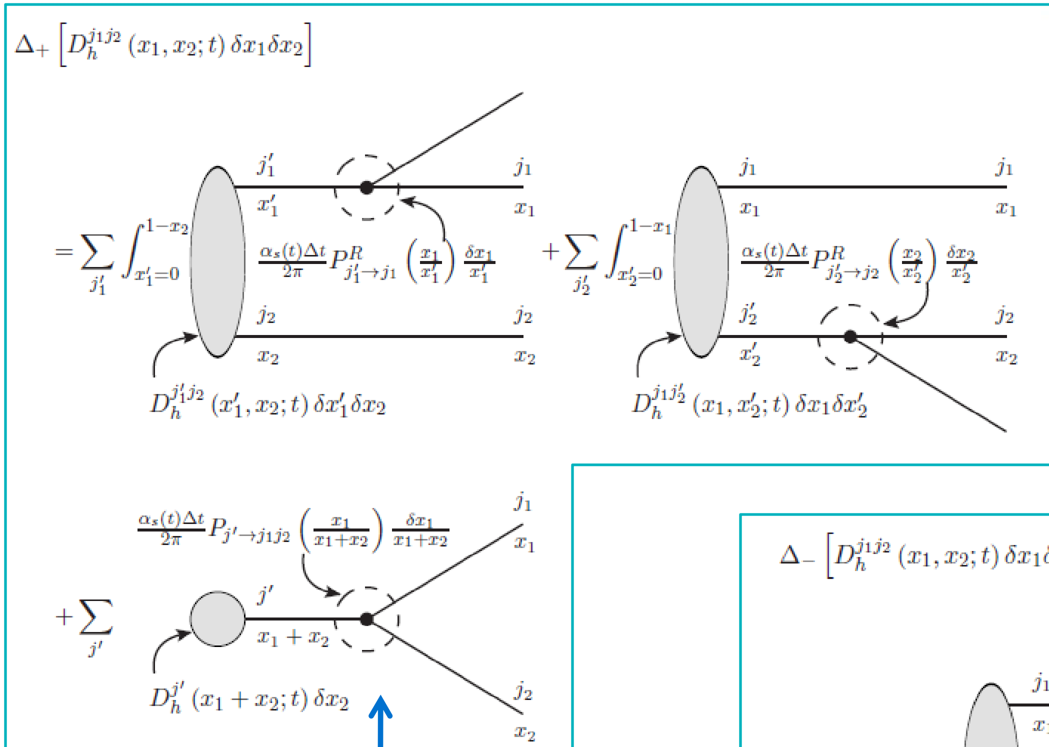
‘1→2’ splitting function

Single PDF

[Structure of last term must be altered at NLO and above]

(Kirschner, Phys.Lett.B84:266, 1979 and Shelest, Snigirev, and Zinovjev, Phys.Lett.B113:325,1982).

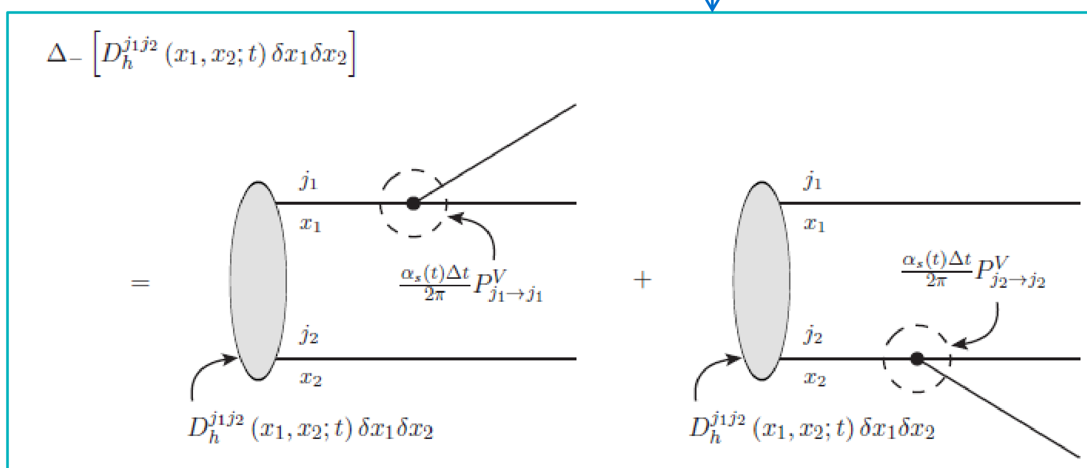
Pictorial representation of double DGLAP equation



Splitting processes acting to increase D^{ij} as the scale is increased from t $\to$ t $+\Delta$t.

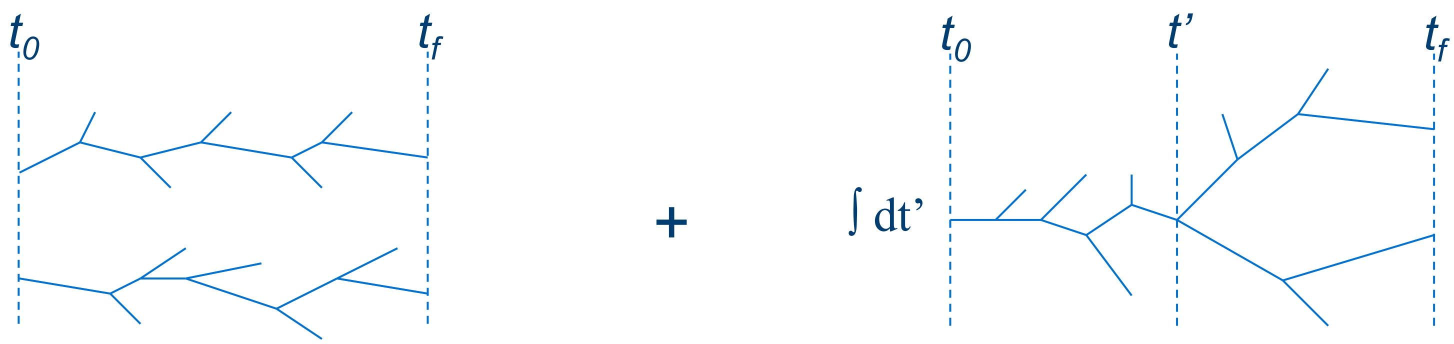
"single PDF feed"

Splitting processes acting to decrease D^{ij} as the scale is increased from t $\to$ t $+\Delta$t.



Double DGLAP evolution as a branching process

Can solve dDGLAP equation to give, schematically:

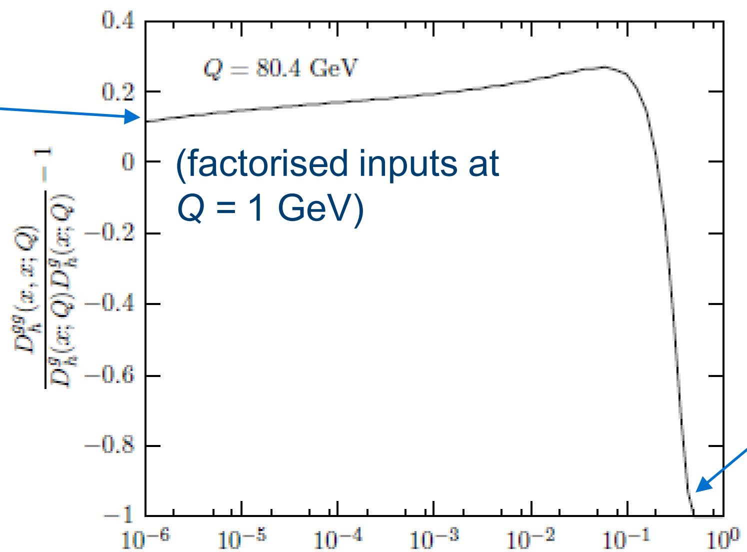


'Independent branching' term – largely preserves factorised forms, except at large *x* where phase space constraints lead to significant deviation from factorised forms.

'Single parton feed' term – extra contribution which causes dPDFs to deviate from factorised forms, particularly at low *x*.

Numerical demonstration that pQCD evolution causes dPDFs to deviate from factorised forms

~10% deviation at low x due to single parton feed contribution



$x_1 = x_2 = x$

pQCD evolution correctly takes account of momentum constraints – ratio of dPDF to factorised form quickly goes to zero on kinematic boundary.

JG and Stirling, 0910.4347, 2009

Input dPDFs

The most accurate approach to modelling the (equal scale) dPDFs is to use the double DGLAP equation along with some suitably chosen inputs at a low scale Q_0.

But what should the inputs look like? **Can we get any theoretical insight**?

First reaction - **NO**! A dPDF at any particular scale receives contributions from **non-perturbative physics**.

The dPDF Sum Rules

Actually – **YES**, we can! We have shown that the following equalities (**sum rule equalities**) are preserved by double DGLAP:

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 D_h^{j_1 j_2}(x_1, x_2; t) = (1 - x_2) D_h^{j_2}(x_2; t)$$

$$\int_0^{1-x_2} dx_1 D_h^{j_{1v} j_2}(x_1, x_2; t) = \begin{cases} N_{j_{1v}} D_h^{j_2}(x_2; t) & \text{when } j_2 \neq j_1 \text{ or } \bar{j}_1 \\ (N_{j_{1v}} - 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = j_1 \\ (N_{j_{1v}} + 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = \bar{j}_1 \end{cases}$$

These equalities are no more than the statements of conservation of momentum and quark number for the dPDFs, and have an interpretation in terms of conditional probabilities.

In general, we expect there to be a hierarchy of such relations:

$$\sum_{j_1} \int dx_1 x_1 D_h^{j_1 j_2 \ldots j_n}(x_1, x_2 \ldots x_n) = \left(1 - \sum_{i=2}^{n} x_i\right) D_h^{j_2 \ldots j_n}(x_2 \ldots x_n)$$

The sum rules impose important constraints on the type of input dPDFs that are allowable ...although non-trivial to implement them!

The Input dPDFs

It is most convenient to work in terms of the 'double evolution' parton flavour basis when trying to construct a set of dPDFs satisfying the sum rules (also when considering dPDF evolution).

In this basis, the parton indices *ij* of a dPDF are one out of the following rather than q_i, g, etc:

A dPDF with one of these indices will be involved in a number sum rule

$$j_v = q_j - \bar{q}_j$$ Valence

A dPDF with one of these indices will be involved in a mtm sum rule

$$\Sigma = \sum \left(q_j + \bar{q}_j \right)$$ Singlet

$$g$$ Gluon

'Tensor' combinations

$$q^+ = q_j + \bar{q}_j$$

$$T_3 = u^+ - d^+$$
$$T_8 = u^+ + d^+ - 2s^+$$
$$T_{15} = u^+ + d^+ + s^+ - 3c^+$$
$$T_{24} = u^+ + d^+ + s^+ + c^+ - 4b^+$$
$$T_{35} = u^+ + d^+ + s^+ + c^+ + b^+ - 5b^+$$

Must choose a set of sPDF inputs to which our dPDF inputs correspond – we use MSTW2008LO inputs (with some slight alterations – e.g. we take $s_v = 0$).

$\Rightarrow$ our input scale Q_0 = MSTW 2008 input scale = 1 GeV.

Can we base our dPDFs on factorised forms?

Wishing to make maximal use of the detailed information we have on sPDFs, and based on popular lore + experimental evidence from the Tevatron, we would like to use input dPDFs which are based around simple products of sPDFs, and become equal to such products in the low *x* limit. Can we do this & still satisfy the sum rules?

Yes – for all dPDFs **except** for the equal flavour valence-valence (EFVV) distributions:

$$\underbrace{\int_0^{1-x_2} dx_1 D_h^{j_v j_v}(x_1, x_2; t_0)} = \underbrace{N_{j_v} D_h^{j_v}(x_2; t_0) - D_h^{j+\bar{j}}(x_2; t_0)}$$

If dPDF is approximately factorised form for small x_1 and x_2, expect $\text{LHS} \sim x_2^{-a_v} \sim x_2^{-0.5}$ for small x_2

Dominated by second term at low x_2 $\Rightarrow \text{RHS} \sim -x_2^{-a_s} \sim -x_2^{-1}$ for small x_2

Take factorised forms as the basis for all other dPDFs, and come back to the problem of EFVV distributions later.

Key features desirable in our inputs

There are two key features that we would like to build in to our set of input dPDFs. These are the following:

1. The dPDFs should be suppressed below factorised values near the kinematical bound $x_1+x_2=1$ due to phase space considerations.
2. Terms should be added/subtracted from certain dPDFs to take account of number effects.

Consider requirement (1) first. In previous studies, universal factors such as $(1-x_1-x_2)$ * or $(1-x_1-x_2)^2$ † multiplying factorised forms have been advocated to take account of phase space effects.

* Das and Hwa, Phys.Lett.B68:459, 1977, Kuti and Weisskopf, Phys.Rev.D4:3418, 1971

† Korotkikh and Snigirev, Phys.Lett.B594:171, 2004

Taking account of phase space effects

Momentum sum rule tells us neither of these options are fully satisfactory. Consider mtm sum rules along the line $x_2 = 0$:

$$\sum_{j_1}\int_0^1 D_h^{j_1 j_2}(x_1, x_2; t) = D_h^{j_2}(x_2; t)$$

Condition perfectly satisfied by factorised dPDFs, but badly violated by dPDFs including a $(1 - x_1 - x_2)$ or $(1 - x_1 - x_2)^2$ factor. A similar argument can be made for the line $x_1 = 0$.

Key requirements of a good phase space factor:

- Primarily responsible for all momentum sum rules being satisfied.
- Must ensure number sum rules are satisfied for those dPDFs not affected by number effects.

Taking account of phase space effects

We use the following phase space factor, which depends on the dPDF parton indices *ij*:

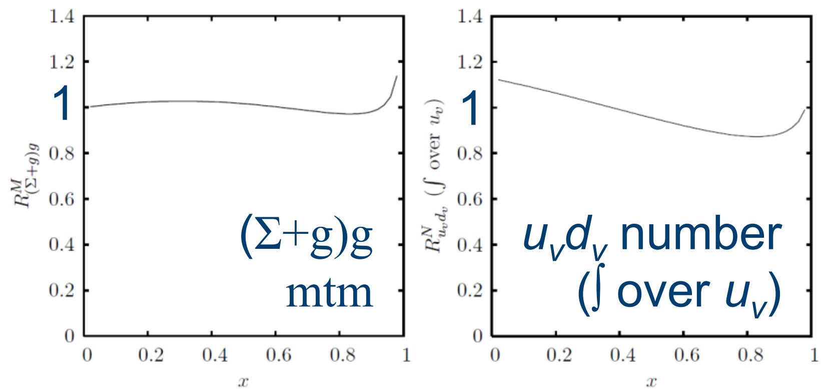
$$\rho^{ij}(x_1, x_2) = (1 - x_1 - x_2)^2 (1 - x_1)^{-2-\alpha(j)} (1 - x_2)^{-2-\alpha(i)} \quad \text{where} \quad \alpha(i) = \begin{cases} 0 & \text{if } i \text{ is a sea parton} \\ 0.5 & \text{if } i \text{ is a valence parton} \end{cases}$$

‘Korotkikh-Snigirev’ factor

Factors compensating for K-S factor drop along $x_1 = 0$ and $x_2 = 0$

ij-dependent factors found to improve extent to which sum rules are satisfied

Relevant sum rules well satisfied with this phase factor:



Taking account of number effects

Number effects can in principle have an impact on any dPDF for which the same parton type appears in both parton indices.

Finite number of valence quarks vs. infinite number of quarks and gluons in the sea ➔ number effects relating to valence quarks are the most important.

e.g.

$$u^+u^+ \equiv (u+\bar{u})(u+\bar{u})$$
$$= (u_v + 2u_s)(u_v + 2u_s) = 2u_s u_v + 2u_v u_s + 4u_s u_s + u_v u_v$$

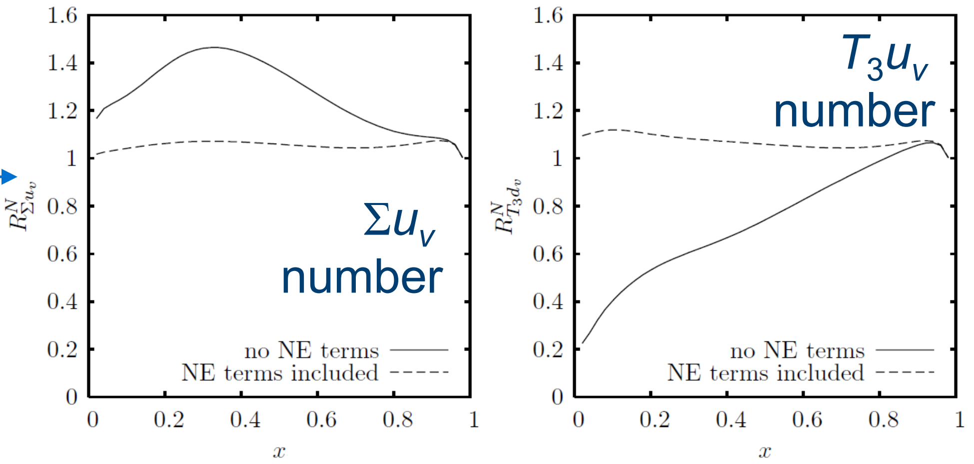
Factorised form × phase space factor reasonable for these terms...

...but not for this part – need to take account of the fact that finding a u_v halves the probability to find another.

Taking account of number effects

In general, can take account of valence number effects in our dPDFs inputs by dividing $u_v u_v$ part of naive factorised forms by 2, and completely removing any $d_v d_v$ piece.

Effect of including number effect terms on the extent to which the affected dPDFs satisfy their sum rules



Equal flavour valence-valence dPDFs

Need to construct suitable EFVV inputs for up, down, **and strange** flavours. Note that our $s_v s_v$ input cannot be zero even though we have taken s_v sPDF zero:

$$\int_0^{1-x_2} dx_1 D_h^{s_v s_v}(x_1, x_2; t_0) = -D_h^{s+}(x_2; t_0)$$

Intuitive explanation: $s_v s_v = ss - \bar{s}s - s\bar{s} + \bar{s}\bar{s}$ and we expect $\bar{s}s, s\bar{s}$ to be slightly larger than $ss, \bar{s}\bar{s}$ due to number effects.

Equal flavour valence-valence dPDFs

Our equal flavour valence-valence inputs are constructed according to the following prescription:

$$D_h^{j_v j_v}(x_1, x_2; t_0) = \frac{N_{j_v} - 1}{N_{j_v}} \underbrace{D_h^{j_v}(x_1; t_0) D_h^{j_v}(x_2; t_0)}_{\text{Naive factorised form}} \rho^{j_v j_v}(x_1, x_2) - 2g^{j\bar{j}}(x_1 + x_2; t_0)$$

Phase space factor

Valence-valence number effect adjustment

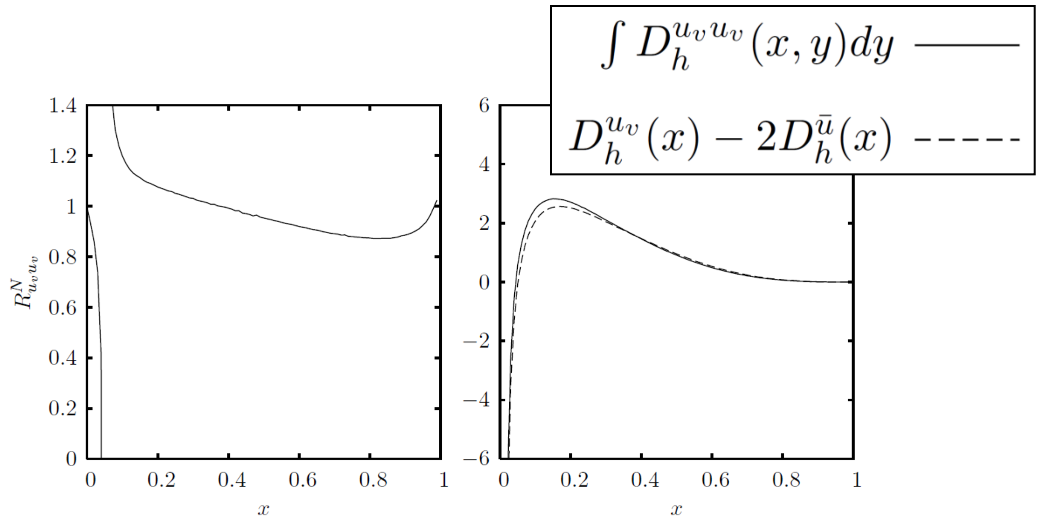
Term to take account of sPDF feed contributions to $-(\bar{j}j + j\bar{j})$ component of dPDF during evolution from some lower scale to t_0.

Inserting the above form into the number sum rule, we find that g must be given by:

$$g^{j\bar{j}}(x; t_0) = -\frac{\partial D_h^{\bar{j}}(x; t_0)}{\partial x}$$

Equal flavour valence-valence dPDFs

This prescription gives $d_v d_v$ and $s_v s_v$ distributions which exactly satisfy their respective sum rules. Extent to which $u_v u_v$ distribution satisfies its sum rule:



Divergence in the sum rule ratio is caused by the integral curve slightly missing a zero in the sPDF quantity it should be equal to.

Final adjustments

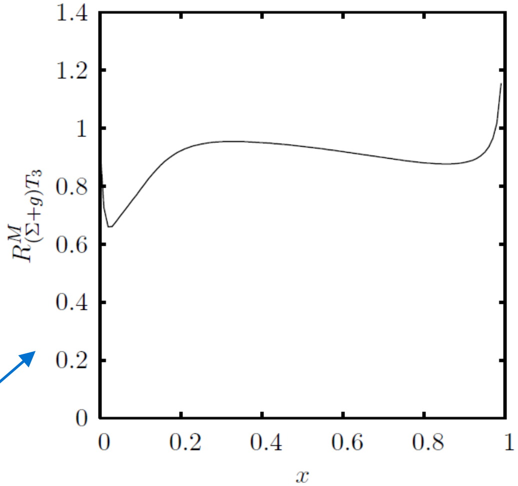
The j^+j^+ distributions contain the same $\bar{j}j + j\bar{j}$ combination that is also found in the EFVV distributions, but here it appears with the opposite sign. For consistency, we should include a contribution to the j^+j^+ dPDFs equal to **plus** $2g^{j\bar{j}}(x_1 + x_2; t_0)$

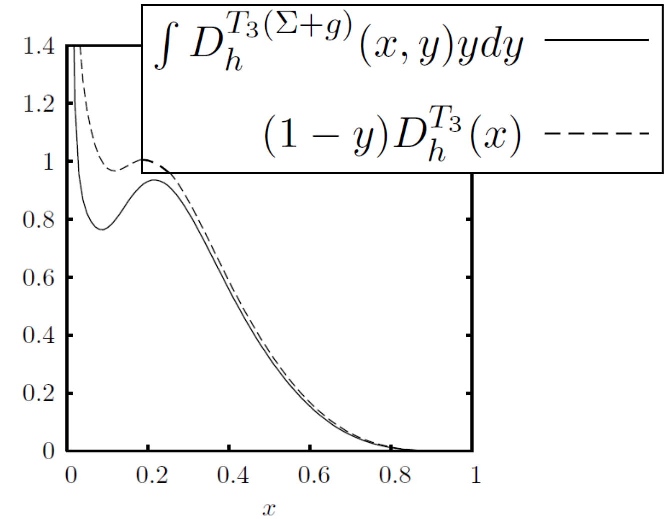
This final adjustment ensures all dPDFs are positive and we see little adverse effect on the extent to which the sum rules are satisfied.

Summary of extent to which our inputs satisfy sum rules

In human flavour basis, **all sum rule ratios within 25% of 1 for *x* < 0.8**

In double evolution basis, similar story, barring trivial divergences. Only exception is $(\Sigma+g)T_3$ mtm sum rule:





The GS09 dPDFs

JG and Stirling, 0910.4347, 2009

Inputs used with numerical implementation of double DGLAP to generate first set of publicly available LO equal-scale dPDFs (available from **HepForge***). Package includes grid of dPDF values spanning $10^{-6} < x_1 < 1$, $10^{-6} < x_2 < 1$, $1\ \mathrm{GeV}^2 < Q^2 < 10^9\ \mathrm{GeV}^2$ + interpolation code.

*http://projects.hepforge.org/gsdpdf/

Comparison of GS09 with factorised dPDFs

JG, Kom, Kulesza, Stirling, 1003.3953, 2010

Comparison in the context of a particular process – **equal sign W pair production.**

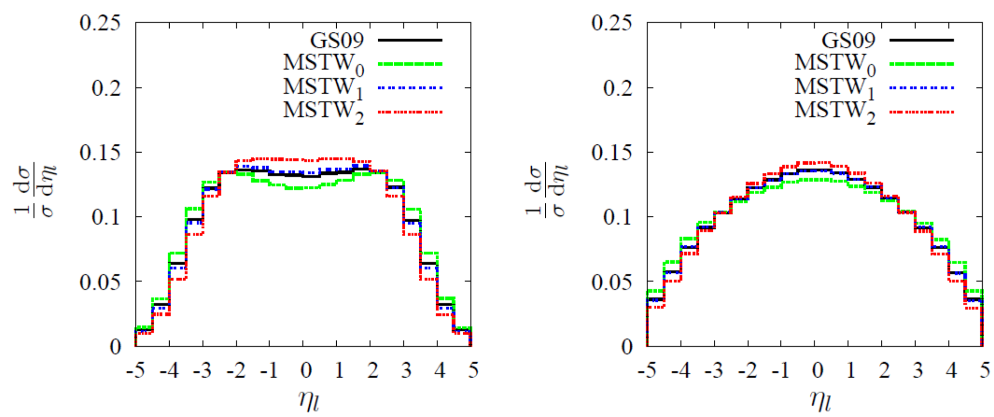
MSTW$_n$ = product of MSTW sPDFs $\times$ $(1 - x_1 - x_2)^n$

Cross sections similar. MSTW$_1$ and MSTW$_2$ sets give smaller cross sections due to $(1 - x_1 - x_2)^{1,2}$ suppression of dPDFs.

$\sqrt{s} = 14$ TeV

	σ_{GS09}	σ_{MSTW_0}	σ_{MSTW_1}	σ_{MSTW_2}
W^+W^-	0.546	0.496	0.409	0.348
W^+W^+	0.321	0.338	0.269	0.223
W^-W^-	0.182	0.182	0.156	0.136
	R			
	0.784	1.00	1.00	1.00

Pseudorapidity distribution of leptons is similar with GS09 and MSTW$_n$ (MSTW$_1$ gives best match).



(a) Positively charged leptons

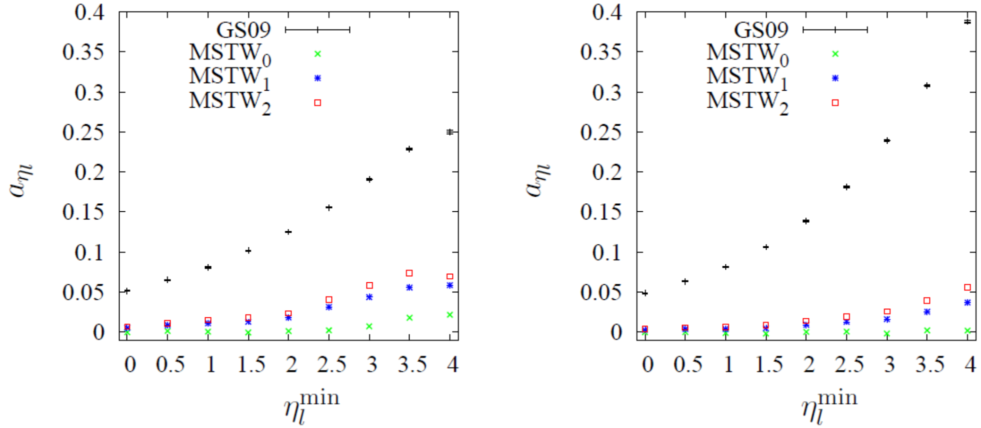
(b) Negatively charged leptons

Lepton Pseudorapidity Asymmetry

However, it is possible to construct physical observables that are sensitive to the correlations inherent in GS09:

opposite hemisphere   same hemisphere

$$a_{\eta_l} = \frac{\sigma(\eta_{l_1} \times \eta_{l_2} < 0) - \sigma(\eta_{l_1} \times \eta_{l_2} > 0)}{\sigma(\eta_{l_1} \times \eta_{l_2} < 0) + \sigma(\eta_{l_1} \times \eta_{l_2} > 0)}.$$



(a) Positively charged leptons

(b) Negatively charged leptons

$a_{\eta l}$ larger for GS09 due to number effect subtractions, especially for large η_l^{min} (i.e. large *x*, where number effect subtractions have the largest impact).

In practice, unlikely to be able to discriminate between GS09 and factorised forms in near future due to backgrounds to same-sign WW (see talk by Steve Kom).

Current Work

Extend treatment to NLO.

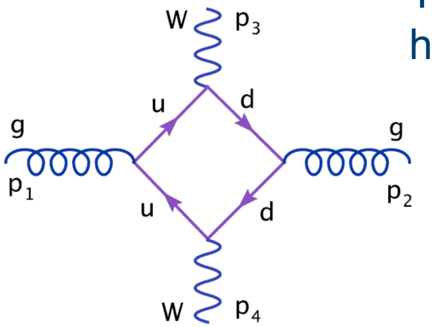
- Need to compute $1 \rightarrow 2$ splitting functions at NLO (trivial at LO).
- Will need NLO coefficient functions for certain benchmark processes (e.g. equal sign WW production).

Possible Issues

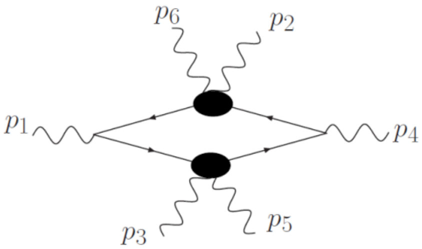
Single parton feed piece of dPDF doesn't 'know' about the size of the proton – how can it be appropriate to assign an effective area σ_{eff} of the same order of magnitude as the size of the proton to this piece?

NB: Same σ_{eff} for all parts of dPDF crucial for sum rules!

Should be able to obtain 1→2 splitting functions by looking at divergent part of diagrams in which one parton splits into two, and then the two daughters each go on to participate in a hard collision. However, several diagrams of this kind have been shown to have no divergences:



T. Binoth et al.
hep-ph/0611170



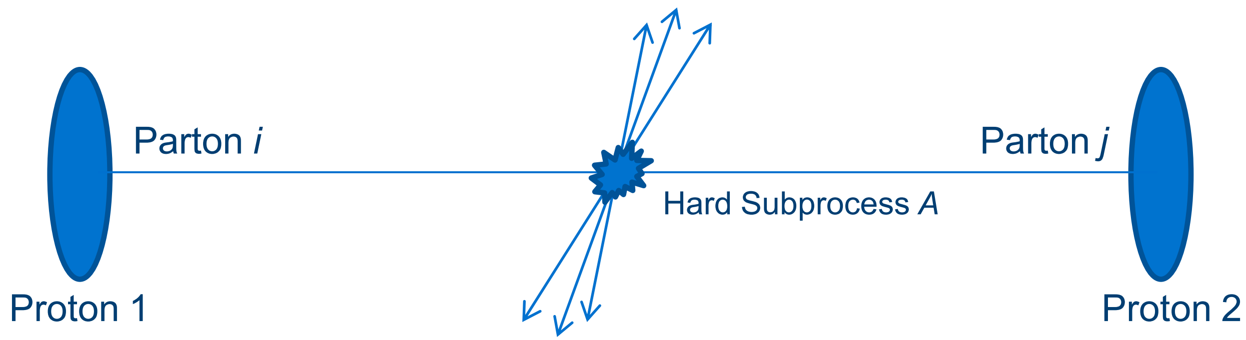
Z. Bern et al.
0803.0494

Summary

- Important to understand DPS – will produce significant backgrounds and interesting signals at the LHC.
- For DPS predictions, require dPDFs. A 'double DGLAP' equation exists dictating the evolution of the equal-scale dPDFs, and we have derived the number and momentum sum rules for these quantities.
- We have produced the first publicly available set of LO equal-scale dDPFs. Sum rules used to guide construction of inputs at Q_0 = 1 GeV, and double DGLAP equation used to obtain dPDF values at other scales.
- Number and momentum correlations in GS09 dPDFs affect the signatures of DPS processes – but may be difficult to see this at LHC due to SPS background.

Backup Slides

Single Parton Scattering



Factorisation theorem ➔ $\sigma_S^{(A)} = \sum_{i,j} \int D_h^i(x_1;Q) D_h^j(x_1';Q) \hat{\sigma}_{ij}^A(x_1, x_1') \, \mathrm{d}x_1 \mathrm{d}x_1'$

(Single) Parton Distribution Functions (sPDFs)

Hard subprocess cross section

Variation of sPDFs with Q is determined by the (single) DGLAP equation:

$$\frac{\partial D_h^i(x;t)}{\partial t} = \frac{\alpha_S(t)}{2\pi} \sum_j \int \frac{dz}{z} D_h^j(z;t) P_{j\to i}\left(\frac{x}{z}\right)$$

(1→1) splitting function

1→2 splitting functions

$\frac{\alpha_S(t)\Delta t}{2\pi} P_{i\to jk}(x)\delta x$ = probability of an *i* parton with mtm 1 splitting to give a *j* parton with mtm *x* and a *k* parton with mtm (1-*x*) when scale is increased from *t* to *t*+Δ*t*

In the above, we have implicitly assumed that a single splitting can only give rise to two particles, such that *jk* carry all mtm of *i* ⇒ 1→2 splitting function with just one mtm argument only makes sense at LO.

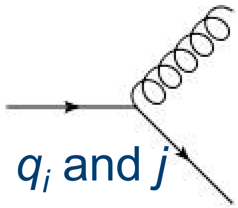
Higher order splitting function must have two mtm arguments:

$$P_{i\to jk}(x_1,x_2) = \delta(1-x_1-x_2)P^{(0)}_{i\to jk}(x_1) + \frac{\alpha_s}{2\pi}P^{(1)}_{i\to jk}(x_1,x_2) + \ldots$$

(& structure of last term in dDGLAP equation must be altered at NLO and above!)

1→2 trivially related to 1→1 splitting functions in LO case.

$$P_{i\to jk}(x) = \begin{cases} P^R_{i\to j}(x) & \text{if } k = \kappa(i,j) \\ 0 & \text{otherwise} \end{cases}$$



κ(*i*,*j*) = only choice of parton that can be combined with *i* & *j* to make a legitimate QCD 3-vertex.

e.g. if $i = q_i$ and $j = g$, $\kappa = q_i$

Equal flavour valence-valence dPDFs

In evolution from t' to t_0 , independent branching terms of dDGLAP equation will only serve to take initial form of an EFVV distribution into its equivalent at t_0. On the other hand, sPDF feed terms will result in an extra contribution appearing in each EFVV dPDF.

Only $-\bar{j}j-j\bar{j}$ component of an EFVV dPDF receives sPDF feed contributions during the evolution – these are of the form:

$$-2\frac{\alpha_s(t)}{2\pi}D_h^g(x_1+x_2;t)\frac{1}{x_1+x_2}\boxed{P_{qg}\left(\frac{x_1}{x_1+x_2}\right)}$$

approx constant

⇒total sPDF feed contribution to EFVV distributions is roughly speaking, just a function of $x_1 + x_2$:

$$D_h^{j_v j_v}(x_1,x_2;t_0) = \frac{N_{j_v}-1}{N_{j_v}}D_h^{j_v}(x_1;t_0)D_h^{j_v}(x_2;t_0)\rho^{j_v j_v}(x_1,x_2) - 2g^{j\bar{j}}(x_1+x_2;t_0)$$

$\int$ $\int$

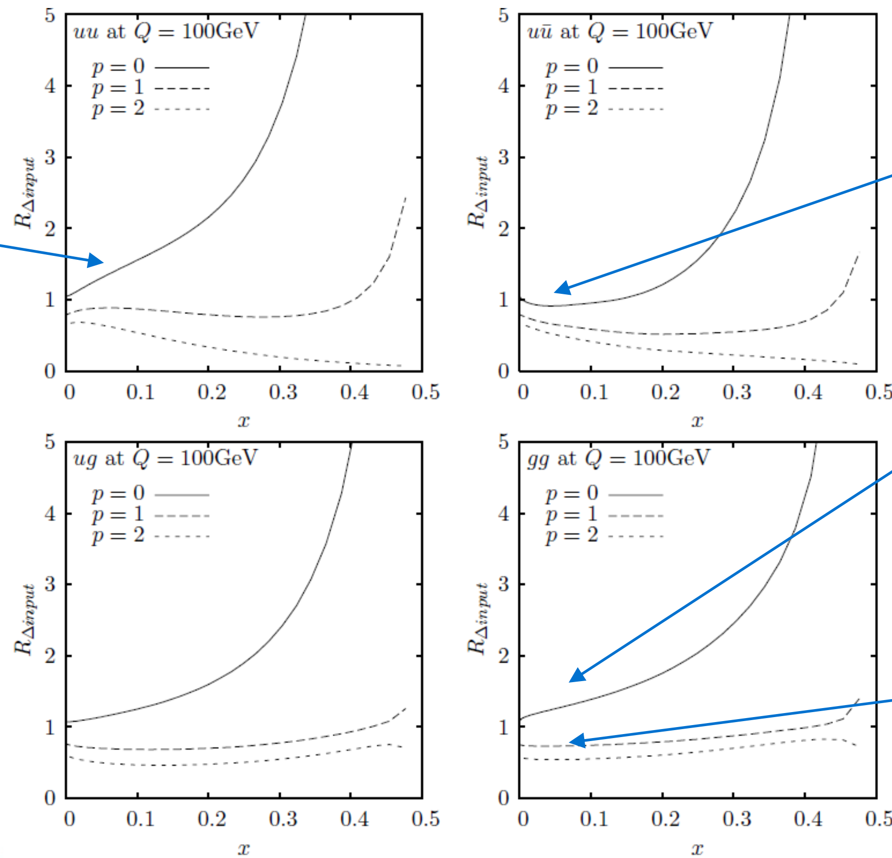
c.f. number sum rule for this dPDF:

$$\int_0^{1-x_2} dx_1 D_h^{j_v j_v}(x_1,x_2;t_0) = (N_{j_v}-1)D_h^{j_v}(x_2;t_0) - 2D_h^{\bar{j}}(x_2;t_0)$$

Effect of change in inputs on Q = *100* GeV dPDFs

$$R^{ij}_{\Delta input}(x;Q) \equiv \frac{D_h^{ij}(x,x;Q)\,\big|_{\text{input } D_h^{ij}(x_1,x_2;Q_0)=D_h^i(x_1;Q_0)D_h^j(x_2;Q_0)(1-x_1-x_2)^p}}{D_h^{ij}(x,x;Q)\,\big|_{\text{input } D_h^{ij}(x_1,x_2;Q_0)=\text{our improved inputs}}}$$

Valence number effect subtractions from GS09 dPDF



Extra $j\bar{j}$ correlation term in GS09 $u\bar{u}$ input

p = 0 factorised input too large at large x – excess filters down to lower x values during evolution. Conversely, p = 1,2 inputs too small at large x – again this filters down during evolution.

Effect of sPDF feed on Q = *100* GeV dPDFs

$$R^{ij}_{\text{no feed}}(x;Q) \equiv \frac{D^{ij}_h(x,x;Q)\,|_{\text{our improved inputs, no sPDF feed}}}{D^{ij}_h(x,x;Q)\,|_{\text{our improved inputs}}}$$

sPDF feed makes approximately universal contribution to all dPDFs at small x (~10%)

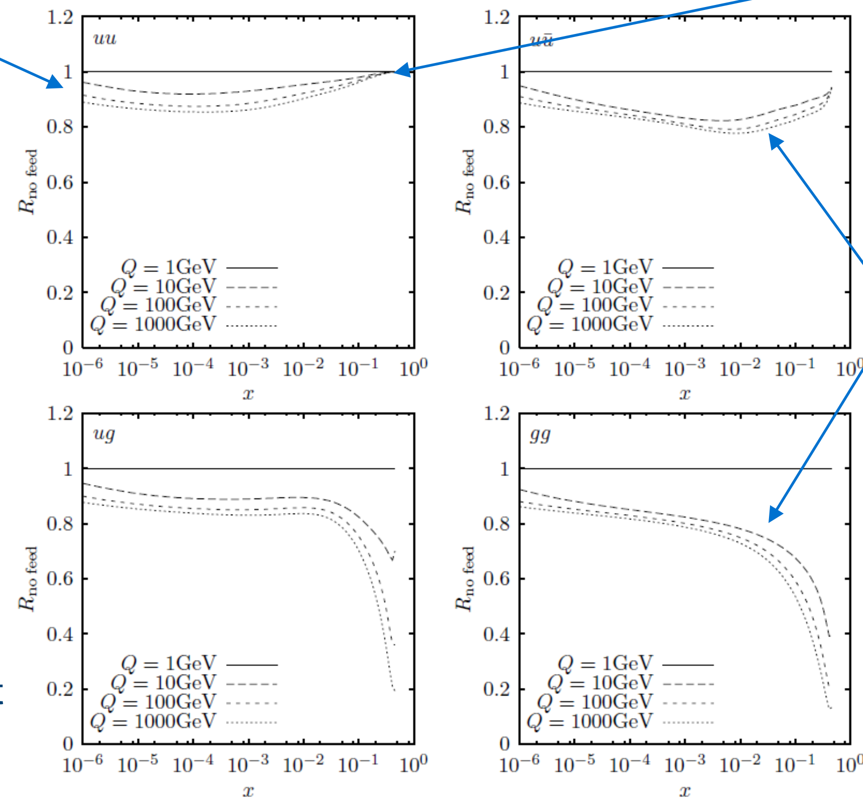
This is because dominant sPDF feed contribution to all dPDFs at low x comes from:

$$g \xrightarrow{1\to 2\text{ splitting}} gg$$

independent branching

ij

No direct sPDF feed

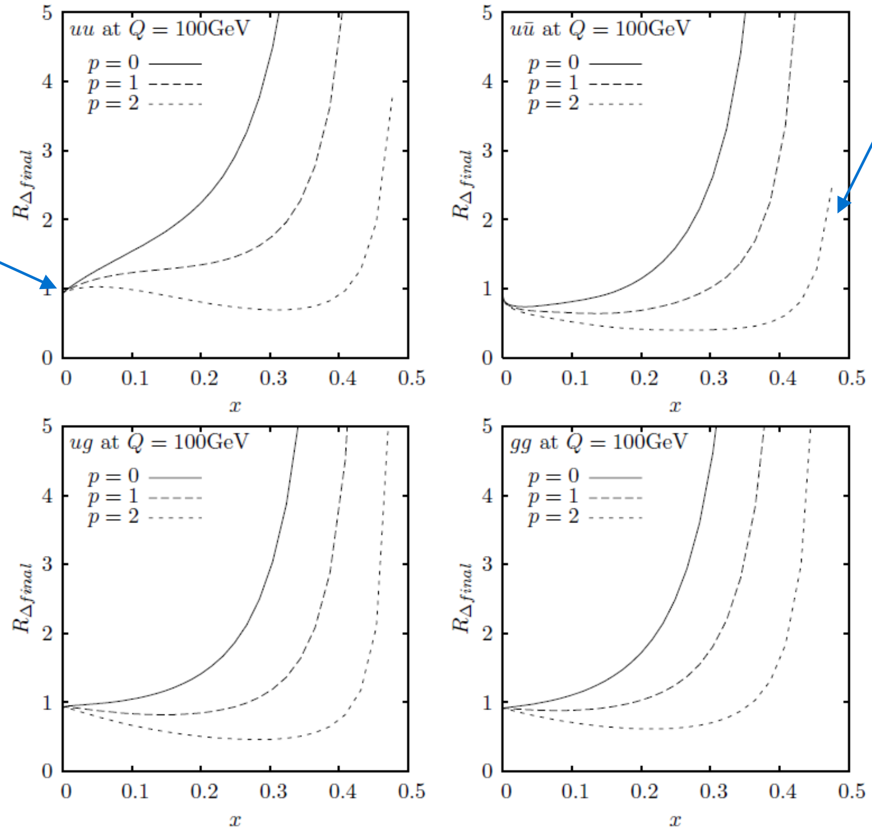


Gluon type evolution pulls PDFs to lower x values a lot more strongly than quark type evolution → dPDFs with gluon indices are more strongly affected at large x by removal of sPDF feed.

Effect of inputs + pQCD evolution on Q = *100* GeV dPDFs

$$R^{ij}_{\Delta final}(x_1, x_2; Q) \equiv \frac{D^i_h(x_1;Q) D^j_h(x_2;Q)(1 - x_1 - x_2)^p}{D^{ij}_h(x_1, x_2; Q)\,|_{\text{our improved inputs}}}$$

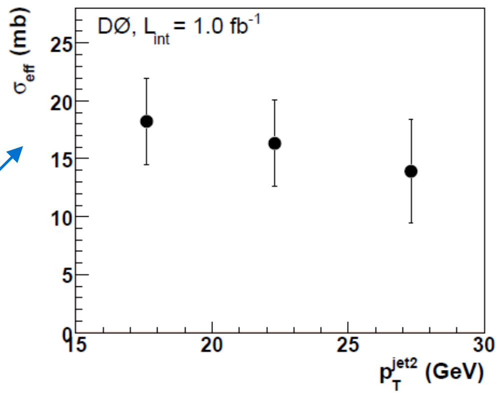
All ratios are about 10% below 1 at small x (lack of sPDF feed in factorised forms)



At Q = 100 GeV, even a $(1 - x_1 - x_2)^2$ suppression factor multiplying factorised forms represents an underestimate in the large x falloff of the dPDFs.

D0 Results

D0 investigated variation of ratio $\sigma_S^{(A)}\sigma_S^{(B)}/\sigma_D^{(A,B)}$ with second largest jet p_T. Data consistent with no variation, although suggestion that ratio decreases with increase in p_T (effects of pQCD evolution on dPDFs?)



Phys. Rev. D81 052012, 2010.

Equal flavour valence-valence dPDFs

Need to construct suitable EFVV inputs for up, down, **and strange** flavours. Note that our $s_v s_v$ input cannot be zero even though we have taken s_v sPDF zero:

$$\int_0^{1-x_2} dx_1 D_h^{s_v s_v}(x_1, x_2; t_0) = -D_h^{s+}(x_2; t_0)$$

Intuitive explanation: $s_v s_v = ss - \bar{s}s - s\bar{s} + \bar{s}\bar{s}$ and we expect $\bar{s}s, s\bar{s}$ to be slightly larger than $ss, \bar{s}\bar{s}$ due to number effects.

Key idea utilised to construct EFVV inputs is the hypothesis that at some scale t' < t_0 only the three valence quarks in the proton may be resolved, and all sea distributions are zero (early GRV idea). At this scale, EFVV dPDFs are given by:

$$D_h^{j_v j_v}(x_1, x_2; t') = \frac{N_{j_v} - 1}{N_{j_v}} D_h^{j_v}(x_1; t') D_h^{j_v}(x_2; t') \widetilde{\rho}^{\, j_v j_v}(x_1, x_2)$$ (= 0 for $j = d$ or s)

Valence-valence number effects.

Phase factor appropriate to scale t'

Equal flavour valence-valence dPDFs

In evolution from t' to t_0 , independent branching terms of dDGLAP equation will only serve to take initial form of an EFVV distribution into its equivalent at t_0. On the other hand, sPDF feed terms will result in an extra contribution appearing in each EFVV dPDF.

Only $-\bar{j}j-\bar{j}j$ component of an EFVV dPDF receives sPDF feed contributions during the evolution – these are of the form:

$$-2\frac{\alpha_s(t)}{2\pi}D_h^g(x_1+x_2;t)\frac{1}{x_1+x_2}\boxed{P_{qg}\left(\frac{x_1}{x_1+x_2}\right)}$$

approx constant

$\Rightarrow$total sPDF feed contribution to EFVV distributions is roughly speaking, just a function of x_1 + x_2:

$$D_h^{j_v j_v}(x_1,x_2;t_0) = \frac{N_{j_v}-1}{N_{j_v}}D_h^{j_v}(x_1;t_0)D_h^{j_v}(x_2;t_0)\rho^{j_v j_v}(x_1,x_2) - 2g^{j\bar{j}}(x_1+x_2;t_0)$$

$\int$ $\int$

c.f. number sum rule for this dPDF:

$$\int_0^{1-x_2} dx_1 D_h^{j_v j_v}(x_1,x_2;t_0) = (N_{j_v}-1)D_h^{j_v}(x_2;t_0) - 2D_h^{\bar{j}}(x_2;t_0)$$

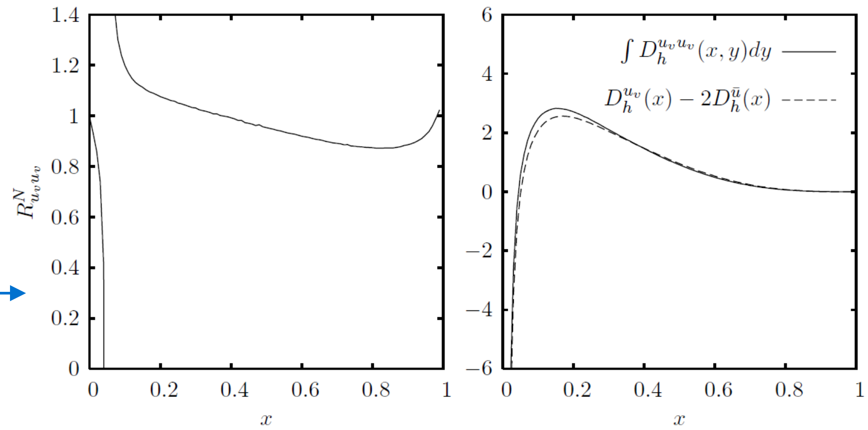
Equal flavour valence-valence dPDFs

$j\bar{j}$ correlation term must satisfy:

$$-2\int_0^{1-x_2} dx_1 g^{j\bar{j}}(x_1+x_2;t_0) = -2D_h^{\bar{j}}(x_2;t_0)$$

This is easy to solve, giving: $g^{j\bar{j}}(x;t_0) = -\dfrac{\partial D_h^{\bar{j}}(x;t_0)}{\partial x}$

This prescription gives $d_v d_v$ and $s_v s_v$ distributions which exactly satisfy their respective sum rules. Extent to which $u_v u_v$ distribution satisfies its sum rule:



Divergence in the sum rule ratio is caused by the integral curve slightly missing a zero in the sPDF quantity it should be equal to.

Numerical Integration of dDGLAP

Estimate of error introduced by numerical integration in an evolution from Q_0 = 1 GeV to Q_f = 100 GeV using 150 points in each *x* direction, and 10 in the *t*.

To produce publicly available grids, 600 points in each *x* direction and 120 in the *t* were actually used.